Abstract — Concepts for sensorless position control of induction motor drives rely on anisotropic properties of the machine rotor. The built-in spatial anisotropy is detected by injecting a high-frequency flux wave into the stator. The resulting stator current harmonics contain frequency components that depend on the rotor position. Models of the rotor saliency serve to extract the rotor position signal using phase-locked loop techniques.

A different approach makes use of the parasitic effects that originate from the discrete winding structure of a cage rotor. It has the merit of providing high spatial resolution for incremental positioning without sensor. The practical implementation of sensorless position identification, and of a high-accuracy position control system are reported.

1. INTRODUCTION

Various concepts for controlled high-performance induction motor drives without speed sensor have been developed in the past few years [1]. Eliminating the speed sensor on the motor shaft represents a cost advantage, which combines favorably with increased reliability due to the absence of this mechanical component and its sensor cable. Speed sensorless induction motor drives are well established in those industrial applications in which persistent operation at lower speed is not considered essential.

Ongoing research has focused on providing sustained operation at high dynamic performance in the very low speed range including zero speed and zero stator frequency. Particularly the zero stator frequency problem continues to attract the interest of many researchers [2]. As of now, a robust solution for general industrial use has yet to be found.

A very recent field of research relates to sensorless position control of induction motors, which can be seen as a more stringent requirement than just maintaining zero speed: it involves zero speed operation at a predetermined rotor position. The absence of a position sensor requires the control algorithm to rely on the anisotropic properties of the rotor.

This paper is an attempt to give an overview on the state of research in this new and challenging technological area.

2. CUSTOM DESIGNED MACHINE ROTORS

2.1 Magnetic Saliency

A custom designed rotor can be given an unsymmetric structure to make it amenable for rotor position detection. One possibility is the periodic variation of the widths of the rotor slot openings as proposed by Lorenz [3]. An example is shown in Fig. 1 for a four-pole squirrel cage rotor. The variation is periodic over one pole pitch. The wider slot openings create flux paths of high magnetic reluctance, and hence a locally low inductance, whereas the narrow openings produce low reluctance paths and a high local inductance. The unsymmetry can be detected from its interaction with a high-frequency flux wave that is originated in the stator.

Such unsymmetric rotor construction justifies the definition of a rotor coordinate system in terms of \( d \)- and \( q \)-axes, where the \( d \)-axis points to that location on the rotor circumference that exhibits the maximum inductance.

3. TRACKING OF ROTOR SALIENCY

3.1 High-frequency machine model

To demonstrate the resulting effect in principle, the behavior of an induction machine having a magnetically salient rotor shall be analyzed in an approximative approach. Ignoring the existence of the fundamental field, only the injection of a high-frequency flux wave of carrier frequency \( \omega_c \) shall be considered. Also the influences of the winding resistance and of magnetic saturation are neglected. The approximation further assumes a restriction to very low speed, \( \omega \approx 0 \). Referring to rotor coordinates (superscript \( \text{R} \)), the stator voltage equation is
\[ u_c^{[R]} = u_c e^{j\phi} = i_{\sigma} \frac{d_i_s}{dt}, \tag{1} \]

where the inductance tensor
\[
i_{\sigma} = \begin{bmatrix} l_{sd} & 0 \\ 0 & l_{sq} \end{bmatrix} \tag{2} \]

is determined by the leakage inductances of both axes, since \( \omega_c >> \omega \). Owing to the magnetic saliency of the rotor, the amplitude of the resulting stator current vector gets periodically modulated. It follows an elliptic trajectory which can be decomposed into two space vectors that rotate in a positive, and a negative direction, respectively:

\[
i_c = \frac{-j u_c}{2 \omega_c l_{sd} l_{sq}} \left( e^{j\phi} - e^{-j\phi} \right) \tag{3} \]

This equation is the analytical solution of (1). The transformation of (3) back to the stationary reference frame

\[
i_c = i_p e^{j\phi} + i_n e^{j(-\omega_c + 2\omega)t} = i_p + i_n \tag{4} \]

shows the existence of a positive sequence current component \( i_p \) of carrier frequency, and a negative sequence component \( i_n \) that rotates at the angular velocity \( -\omega_c + 2\omega \). The latter component can be processed to extract the angular position \( \phi = \omega t \) of the rotor.

The trajectory of the current vector \( i_c \) is in fact an ellipse, the angular inclination of which equals the rotor position angle. As this signal is buried under the much larger fundamental current, its extraction is problematic.

### 3.2 Rotor position detection

The information on the rotor position angle \( \phi = \omega t \) is contained in the high-frequency negative sequence component of the stator currents, as expressed by the second term in (4). A signal flow structure to extract this information is shown in Fig. 2. The upper portion shows the induction motor, which is fed from a pulsewidth modulated PWM inverter. An additional carrier signal \( u_c = u_c \exp(j\phi) \) is added to the reference voltage vector \( u_s^* \) that is provided by the regular motor control [3]. This signal creates a balanced high-frequency three-phase voltage system across the motor terminals, the phase sequence of which is positive. The motor current \( i_c \) then exhibits additional components of carrier frequency that need to be suppressed by the low path filter \( \text{LPF1} \) to obtain the feedback signal \( i_{\text{d1}} \) for the current control system. The phase lag of this filter influences the controller design. It impairs the dynamic performance of the current control loop.

Current components of carrier frequency are separated from the measured current signal by a bandpass filter \( \text{BPF} \). This filter almost eliminates the fundamental current and attenuates the switching harmonics of the PWM inverter. Its output signal \( i_c \) is composed of a positive sequence and a negative sequence current component, both having carrier frequency \( \omega_c \).

Owing to the saliency of the machine, the negative sequence component \( i_n \exp(2j\omega_c - \omega_c + \phi) \) in (4) contains the desired position information, since \( 2\omega = \omega_t \) is the double rotor position angle. This signal is extracted from the measured stator currents using a phase-locked loop (PLL) arrangement as shown in the lower portion of Fig. 2. The estimated rotor position angle \( \hat{\phi} \) is obtained as the output of the PLL. It serves to generate a reference vector \( r = \exp(2j\hat{\phi} - \omega_c \hat{\phi}) \) of unity amplitude and negative phase sequence. This signal is locked by the PLL onto the rotor position dependent negative sequence component \( \hat{i}_n \) of the filtered stator current signal \( \hat{i}_c \). To achieve this, the phase difference between the two signals is detected by taking the vector product of the respective space vectors. Its \( z \)-component \( \hat{\epsilon} = r \times \hat{i}_n \) approaches zero when the two vectors rotate at the same frequency and assume zero phase displacement. The zero condition indicates that the estimation of the rotor position angle is correct: \( \hat{\phi} = \phi \).

The PLL structure shown in the lower portion of Fig. 2 ensures that the zero condition is maintained in the time average. It consists of the lowpass filter \( \text{LPF2} \), a PID controller, and a model of the mechanical subsystem of the drive, and a saliency.
The saliency model serves to impress on the reference vector \( \hat{r} \) the same rotor position dependent variation that the real machine, through its saliency, forces on the negative sequence current component. The saliencies shown in Fig. 1 exhibits two full cycles per pole pair, i.e. per rotor revolution of the representative two-pole machine. Hence the corresponding negative sequence current in (4) is

\[ i_n = i_t \exp(-j \omega_t + 2 \delta), \]

where \( \omega_t = \delta \). A signal of the same frequency is generated by the saliency model based on the estimated rotor position \( \hat{\delta} \); this signal is the reference vector \( \hat{r} \).

It shall be recalled that the band-passed current signal \( i_c \) is composed of two vector components having a negative and a positive direction of rotation, respectively. Only the negative sequence component \( i_n \) contributes to the angular position error \( \epsilon \), while the positive sequence component \( i_p \) creates superimposed oscillations of double carrier frequency. These disturbances are removed by the low-pass filter LPF2. The filtered error \( \epsilon_t \) is then a dc signal in principle. It feeds the PID controller, the output of which acts as a torque signal on the mechanical system model. The output assumes a positive value when the estimated angular rotor position lags the negative sequence current \( i_n \). In this case, the controller generates an accelerating torque component \( \hat{T}_{cl} > 0 \) which tends to correct the phase lag error. The estimation dynamics are improved by the feedforward signal \( \hat{T}_L \) which represents an estimate of the load torque.

### 3.3 Design considerations

The selection of the carrier frequency is a trade-off between several concerns. The interaction of the carrier wave with the rotor saliency takes place just below the rotor surface. Hence the carrier frequency must be high enough to create a deep bar effect that prevents the high-frequency flux wave from substantially linking with the rotor bars; it must be low enough such that the skin effect in the rotor laminations does not repel the flux from penetrating below the rotor surface [3]. The deep bar effect confines the carrier frequency to the range beyond 100 Hz, while the lamination skin effect, responsible for the upper frequency bound, starts beyond 400 Hz, depending on the lamination thickness. These constraints permit little freedom for the choice of the carrier frequency. The close frequency bounds on either side of the carrier contribute to making the extracted position information a weak signal.

Other concerns relate to the separation in spectral bandwidth between the fundamental frequency, the carrier frequency, and the inverter switching frequency, respectively. The bandwidth separation between these frequency components must be sufficiently large to allow their separation by lowpass and bandpass filtering. If field weakening is included, the fundamental frequency ranges up to about 200 Hz. The switching frequency depends on the power rating of the drive. While switching frequencies up to 10 kHz are common at lower power levels, the switching frequency reduces considerably as the power increases.

### 3.4 Multiple saliencies

As an extension of his general approach to utilize rotor anisotropies for sensorless position detection, Lorenz has investigated the possibilities to exploit the inherent parasitic saliencies of a cage rotor [4]. These relate to slot effects, rotor eccentricity, magnetic saturation, and manufacturing variations. Using an injected high-frequency flux wave as a means to detect these saliencies produces negative sequence current vectors of multiple frequencies. Each frequency component is related to a certain saliency. Nonsinusoidal saliencies produce corresponding harmonics of the respective negative sequence current.

Lorenz analyzes a machine having a single, sinusoidally distributed saliency with a period equal to a pole pitch [4]. The machine has a custom designed rotor where sinusoidally modulated rotor slot widths introduce a magnetic saliency. The rotor construction is shown in Fig. 1. The discrete nature of the rotor slots themselves represents another saliency. The frequency of this parasitic saliency is 14 times the frequency of the primary saliency in a 4-pole machine \( (2p = 4) \) having \( N = 28 \) rotor slots.

The resulting high-frequency negative sequence current is

\[
i_n = i_2 e^{-j(\omega_2 t - \varphi_2)} + i_{slot} e^{-j(\omega_l t - \varphi_{slot})} + i_{u} e^{-j(\omega_u t - \varphi_u)}.
\]

It consists of three components. The first component \( i_2 \) relates to the custom designed rotor saliency of two cycles per pole pair. The nature of this component has been discussed in a previous Section 2.

The second current component \( i_{slot} \) is created by rotor slotting, a saliency that exhibits \( N/p \) cycles per pole pair, which number multiplies with the rotor speed and then determines the offset from the carrier frequency \( \omega_2 \); \( \varphi_{slot} \) is the corresponding phase displacement angle.

The third negative sequence current component is \( i_u \). This component is generated by winding unsymmetries, and by unbalances in the stator current acquisition circuits.
Although such unbalances are always present in a practical implementation, they have negligible effects in a standard current control system. They do exert an influence on the saliency induced stator currents as these are about three orders of magnitude lower than the fundamental current. Small unbalances in the positive sequence system of carrier frequency, as measured for signal processing as a component of the total stator current, create the negative sequence current \( i_n \) in (5).

The characteristics of this current signal are best understood when looking at its trajectory in a reference frame that rotates in a negative direction at the frequency \(-\omega_c + 2\omega\) of the dominant saliency. If the unbalance current \( i_u \) is neglected, (5) becomes

\[
i_n^{[\omega_c-2\omega]} = i_2 e^{-j\omega_2} + i_{\text{slot}} e^{-j(\frac{N}{p-2})\omega_2 + \phi_{\text{slot}}}.
\]  

(6)

The two components of this transformed current are visualized in the vector diagram Fig. 3. Here, the stationary reference frame appears rotating at \(-\omega_c - 2\omega\) in a positive direction. The dominant current component \( i_2 \) is then a constant vector. It defines a center point around which the vector of the slot harmonic rotates at the angular velocity \((N/p - 2)\omega\) in a positive direction. The rotor position information is contained in the constant current vector \( i_2 \), while the rotating vector \( i_{\text{slot}} \) that originates from the parasitic slotting effect acts as a disturbance.

The resulting negative sequence vector \( i_n \), as seen in the \(- (\omega_c + 2\omega)\)-coordinate system, exhibits a pulsating phase displacement with respect to the dominant vector \( i_2 \), and its magnitude varies periodically. The phase angle oscillations constitute a severe disturbance of the acquired rotor position signal. Both the phase angle and magnitude oscillations endanger the local stability of the PLL, [4]. Also the dynamic tracking capability of the PLL is impaired as the magnitude variations reflect directly on the closed loop gain.

3.5 PLL with multiple saliency model

A PLL having a multiple saliency model in its feedback channel is proposed as a solution to the aforementioned problems. The signal processing structure is shown in Fig. 4. The presence of multiple negative sequence current harmonics in the acquired current signal \( i_n \) makes their separation from the positive sequence harmonics more critical, which calls for better filtering techniques. For this purpose, the measured current signal \( i_n \) is transformed to the +\(\omega_c\)-coordinate frame, in which the positive sequence carrier component appears as a dc current. It is suppressed by a highpass filter. The filter time constant \( \tau_s \) is selected such that also the transformed fundamental current gets sufficiently attenuated.

The filtered current is subsequently transformed to the \(-\omega_c\)-reference frame. A lowpass filter (not shown in Fig. 4) removes the switching harmonics. The resulting signal \( i_2 \) contains all negative sequence components. It forms the input to the PLL. Other than the single saliency approach in Fig. 2, which operates in stationary coordinates, the PLL of this concept processes the negative sequence signals in negative sequence carrier coordinates; all frequencies are therefore transposed to a lower band:

\[
i_n^{[-\omega_c]} = i_{2y} [2\omega r + \phi_2] + i_{\text{slot}} [N/p \omega r + \phi_{\text{slot}}] + i_u e^{j\phi_u}.
\]  

(7)

Both the custom rotor saliency and the slotting effects are included in the saliency model that generates the reference vector from the estimated rotor position angle \( \hat{\delta} \) as

\[
r = i_2 e^{j(2\hat{\delta} + \phi_2)} + i_{\text{slot}} e^{j(N/p \hat{\delta} + \phi_{\text{slot}})}
\]  

(8)

With all model and controller parameters properly tuned, the reference vector follows the same trajectory, Fig. 2, as the negative sequence current \( i_n \). Both vectors are in exact synchronism if the rotor position is accurately tracked, \( \hat{\delta} = \omega r \), since the dc offset vector \( i_u \) in (5) caused by unbalances is being subtracted from the negative sequence current vector \( i_n \).
The use of a multiple saliency model eliminates the disturbances due to multiple frequency tracking; the amplitude variation of the error signal $\varepsilon$ persists, though. More than that, it is squared when the vector product $\varepsilon = r \times i_n$ is taken, which heavily influences upon the closed loop gain of the PLL.

3.6 Compensation of saliencies

An alternative is the approach shown in Fig. 5. Only one of the existing saliencies is incorporated in the saliency model while the influence of other saliencies on the negative sequence current is eliminated from the PLL input. This approach is seen as the preferred solution in [4]. The PLL tracks only a single frequency. Hence the error signal $\varepsilon$ is free from high-frequency oscillations, and the closed loop gain of the PLL is constant.

3.7 Saturation induced saliencies

The magnetic saturation caused by the fundamental flux wave will also affect the magnetic resistance of those outer portions of the rotor teeth which serve as the flux paths for the high-frequency leakage fields. The effect can be interpreted as a saturation induced saliency. This saliency displaces at slip frequency with respect to the rotor. It creates additional positive and negative sequence currents of near carrier frequency, adding another term

$$i_{\text{sat}} = i_{\text{sat}} e^{-j((\omega_c - 2\omega_s)t - \phi_{\text{sat}})}$$

(9)

to the space vector of the negative sequence currents in (5). The frequency $-(\omega_c + 2\omega_s)$ of this current vector is in very close neighborhood to the frequency $-(\omega_c + 2\omega)$ of the position dependent current components, since $\omega = \omega_c$. A separation may be difficult. The problem has not yet been addressed in the literature.

3.8 Results and discussion

Experiments were reported using a 4-pole machine having 28 rotor slots. The rated machine current is 120 A. The machine was fed from a 12 V (twelve volts) dc bus. A 250-Hz carrier was injected for position tracking using 15% of the dc bus voltage. The authors comment that all parameters of the saliency model, the machine model, and of the PLL have been properly tuned before the measurements were taken, and that the test was conducted at zero fundamental current.

The following negative sequence currents, referred to the rated current, were measured:

- unbalance component $i_u = 0.27$
- rotor saliency $i_2 = 0.22$
- slotting saliency $i_{\text{slot}} = 0.069$

The trajectories of the respective negative sequence current components are shown in Fig. 6 as reproduced from [4]. They are referred to the $-\omega_c$-coordinate system.

A graphic representation of the respective current com-
ponents and their spectral relationship are shown in Fig. 7. The operating range is assumed as 0 ... 10 Hz which corresponds to 0 ... 300 rpm. Higher speed values possibly entail frequency separation problems. The frequency spread of the respective saliencies is indicated in Fig. 7 by the two shaded bars, the heights of which represent the current amplitudes.

The accuracy of the position estimation is illustrated by the graph Fig. 8.

A major problem of the approach can be seen in the extrem low signal level. The signal-to-noise ratio obtained is less than $10^{-3}$ which requires the use of high resolution A/D-converters in a practical system. The effect of inverter dead time on the high-frequency carrier waveform has not been analyzed, nor has a current control system been implemented. It is expected that the dynamic performance of the current control system is degraded by the lowpass filter in the feedback path, Fig. 2. A PLL using a double saliency model requires the proper tuning of 10 independent parameters.

The aforementioned condition of zero stator current leaves the motor without excitation and torque; it acts solely as a passive position sensor. Another experiment is reported in which the machine was successfully operated as a motor and position sensor at 5% rated torque. The torque was held constant while a speed variation of $\pm 50$ rpm ($\pm 0.83$ Hz) was enforced by an external dynamometer.

Although no comments are given by the authors, it is assumed that magnetic saturation inhibits operation at higher fundamental current. Saturation induced negative sequence currents of near carrier frequency are difficult to separate from the rotor position induced components, as their respective frequencies, $-\omega_c + 2\omega_s$ and $-\omega_c + 2\omega_s$, are in close neighborhood since $\omega_c \approx \omega_s$. It may be a viable solution to discard the concept of custom designed rotors with built-in saliency and have the PLL synchronized on the rotor slot induced current signals instead. These signals, although smaller in magnitude, have a better separation in bandwidth from the saturation induced signals, and

![Fig. 8: Rotor position estimation using a saliency model and a compensator for a machine having two saliencies; (reproduced from the original curves in [5])](image)

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![Fig. 9: Linear representation of an induction motor having 18 stator slots and 28 rotor slots; (a) phase winding $a$ and rotor slots, (b) resulting flux density distribution](image)

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![Fig. 10: Simplified induction motor having only 2 rotor bars](image)

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![Fig. 11: Distributions in a 2-slot machine with only phase $a$ energized; (a) flux density distribution, (a) location of rotor slots, (c) mutual inductance between stator and rotor winding, (d) total leakage inductance of stator winding phase $a$](image)
their isolation can be easier performed.

4. STANDARD INDUCTION MACHINES

A position sensing concept that does not rely on a custom designed machine rotor was proposed by Jiang [5]. In his approach, the rotor position signal is extracted from the machine voltages.

To analyze the related phenomena, the discretized winding structure of an induction machine is studied. As an example, Fig. 9(a) illustrates the situation for an induction motor having 18 slots in the stator and 28 slots in the rotor. Only phase \(a\) of the stator winding is considered in a first step, while the other phase currents are assumed zero. The phase current \(i_{sa}\) produces a flux density distribution \(B_a(\phi)\) that is displayed in Fig. 9(b).

4.1 Simplified rotor winding

To approach the problem in a simplified way, the cage rotor is considered to have only two rotor bars as shown in Fig. 10. The total leakage inductance \(l_{s\sigma a}\) is defined by

\[
u_{sa} = l_{s\sigma a} \frac{dl_{sa}}{d\tau},
\]

assuming that the mechanical rotor speed equals zero. The mutual inductance between the distributed phase \(a\) stator winding and the rotor winding that is formed by the two rotor bars is

\[
m_{s1} = \frac{\delta + \pi/2}{\delta - \pi/2} B_a(\alpha) l d\alpha,
\]

where \(l\) is the axial length of the rotor. The total leakage inductance is derived from (11) as

\[
l_{s\sigma a} = l_s \left(1 - \frac{m_{s1}^2}{m_{s1}^2} l_{1}\right)
\]

where \(l_s\) is the inductance of the stator winding and \(l_1\) is the inductance of the single rotor winding.

Fig. 11(b) shows the simplified rotor winding at an angular displacement \(\delta\) against the \(\alpha\)-axis of the stator winding. The mutual inductance \(m_{s1}\) varies with the displacement angle \(\delta\) as shown in Fig. 11(c). Fig. 11(d) shows that the total leakage inductance \(l_{s\sigma a}\) as defined by (11) exhibits a distinctive dependency of the rotor displacement angle \(\delta\).

4.2 The 4-slot machine

The problem is further developed by introducing two more bars to the rotor as shown in Fig. 12. The resulting system of three intercoupled windings is described by

\[
u = \left( R + L(\delta) \frac{d}{d\tau} + G(\omega) \right) i
\]

where

\[
u_{sa} = \begin{bmatrix} u_{sa} \\ 0 \\ 0 \end{bmatrix}, \quad i = \begin{bmatrix} i_{sa} \\ i_1 \\ i_2 \end{bmatrix}
\]

The matrix \(R\) represents the winding resistances. The angular velocity \(\omega\) is again assumed to be zero, and hence \(G(\omega)\) is zero. The inductance matrix of the machine

\[
L = L_2 = \begin{bmatrix} l_s & m_{s1} & m_{s2} \\ m_{s1} & l_1 & m_{12} \\ m_{s2} & m_{12} & l_2 \end{bmatrix}
\]

contains the inductance matrix

\[
L_{s\sigma 2} = \begin{bmatrix} l_1 & m_{12} \\ m_{12} & l_2 \end{bmatrix}
\]

of the two rotor windings as a subset.

To compute the total leakage inductance

\[
l_{s\sigma a} = \frac{\nu_{sa}}{dl_{sa}/d\tau}
\]
from (10), the matrix equation (13) is solved for

$$\frac{d i_{ka}}{d \tau} = u_{sa} \frac{\det L_{12}(\delta)}{\det L_2(\delta)}$$

(19)

which is inserted to (18) to yield

$$l_{\sigma a}(\delta) = \frac{\det L_2(\delta)}{\det L_2(\delta)}$$

(20)

The two rotor windings in Fig. 12 are symmetric and orthogonal, hence

$$l_1 = l_2$$

and

$$m_{12} = 0,$$

(21)

which leads to the desired solution for the total leakage inductance

$$l_{\sigma a}(\delta_N) = l_s \left(1 - \frac{m_{s1}^2(\delta) + m_{s2}^2(\delta)}{l_1 l_2} \right)$$

(22)

The respective mutual inductances $m_{s1}(\delta)$ and $m_{s2}(\delta)$ are the orthogonal functions shown in Fig. 13(a). They enter with their squares into (22), thus introducing a multiplication factor $N$ between the spatial frequency of $m_{s1}(\delta)$ and $m_{s2}(\delta)$ on one hand, and the spatial frequency of $l_{\sigma a}$ on the other. The factor $N$ expresses exactly the number of rotor bars. This justifies introducing a new spatial variable $\delta_N = N\delta$ to characterize the periodicity of variation of the total leakage inductance. This variation is shown in Fig. 13(b).

### 4.3 The general induction motor

A general induction motor may have $N$ rotor bars incorporated in its squirrel cage. There are $1/2 (N^2 + N)$ mutual terms in the inductance matrix

$$L_N = \begin{bmatrix}
    l_s & m_{s1} & m_{s2} & \cdots & m_{sN} \\
    m_{s1} & l_1 & m_{s2} & \cdots & m_{sN} \\
    m_{s2} & m_{s1} & l_2 & \cdots & m_{sN} \\
    \vdots & \vdots & \vdots & \ddots & \vdots \\
    m_{sN} & m_{s1} & m_{s2} & \cdots & l_N
\end{bmatrix}$$

(23)

which greatly increases the periodicity of variation of the total leakage inductance. The submatrix of the rotor windings in Fig. 12 are symmetric and orthogonal, hence

$$l_1 = l_2$$

and

$$m_{12} = 0,$$

(24)

The total leakage inductance

$$l_{\sigma a}(\delta_N) = \frac{\det L_{N}(\delta_N)}{\det L_{1N}(\delta_N)}$$

(25)

varies with the rotor position angle $\delta_N$ as shown in Fig. 14 for the case $N = 28$, which refers to the machine design of Fig. 9. The curve in Fig. 14 represents one half of a revolution of the motor shaft.

The total leakage inductance curves of a real machine do not exhibit the very detailed structures of Fig. 13 as the widths of the stator and rotor slots are finite and the rotor...
The equivalent circuit of the motor is shown in Fig. 15. It represents a particular set of phase potentials at the machine terminals. When the switching state vector $\mathbf{u}_1$ is active, the three motor terminals are forced to the respective potentials $u_a = U_d/2$ and $u_b = u_c = -U_d/2$, or $(+- -)$ as symbolically indicated in Fig. 15.

The equivalent circuit of the motor is shown in Fig. 16(a). The following equations hold

$$U_d = l_{\sigma a} \frac{di_a}{d\tau} + u_{ia} - l_{\sigma b} \frac{di_b}{d\tau} - u_{ib} \quad (26)$$

$$U_d = l_{\sigma a} \frac{di_a}{d\tau} + u_{ia} - l_{\sigma c} \frac{di_c}{d\tau} - u_{ic} \quad (27)$$

$$i_a + i_b + i_c = 0 \quad (28)$$

$$u_{ia} + u_{ib} + u_{ic} = 0 \quad (29)$$

Time is normalized in these equations: $\tau = \omega_R t$, where $\omega_R$ is the nominal stator frequency. The measured phase voltages $u_a$, $u_b$ and $u_c$ are summed up to form

$$u_{\sigma} = u_a + u_b + u_c. \quad (30)$$

Using the definitions in Fig. 16(a) and referring to (28) and (29) we have

$$u_{\sigma} = \left( l_{\sigma a} - l_{\sigma c} \right) \frac{di_a}{d\tau} + \left( l_{\sigma b} - l_{\sigma c} \right) \frac{di_b}{d\tau}. \quad (31)$$

The derivatives $di_a/d\tau$ and $di_b/d\tau$ are eliminated from this equation using (26), (27) and (28):

$$u_{\sigma}^{(1)} = \frac{U_d \left( l_{\sigma a} + l_{\sigma c} - 2l_{\sigma b} \right) + \left( -u_{ia} \right) \left( l_{\sigma a} + l_{\sigma c} - 2l_{\sigma b} \right) + \left( -u_{ib} \right) \left( l_{\sigma b} + l_{\sigma c} - 2l_{\sigma a} \right) + \left( -u_{ic} \right) \left( l_{\sigma c} + l_{\sigma b} - 2l_{\sigma a} \right)}{l_{\sigma a} + l_{\sigma b} + l_{\sigma c}} \quad (32)$$

The left-hand side of (32) is marked by a superscript so as to refer to the particular inverter switching state $\mathbf{u}_1$ on which this analysis is based. Such switching state will periodically occur during normal operation of the PWM controlled inverter. If this happens, the three phase voltages $u_a$, $u_b$ and $u_c$ are sampled and summed to compute $u_{\sigma}^{(1)}$ as per (32), which value is then temporarily stored.

To complete the measurement procedure, the original duration of $\mathbf{u}_1$ as commanded by the pulsewidth modulator is extended for a very short additional time interval $\Delta \tau_0$. The extension introduces a flux linkage error $u_1 \Delta \tau_0$ which is subsequently compensated by an additional change of flux linkage $u_4 \Delta \tau_0$, where $u_4 = -u_1$, i.e. the respective opposed switching state $\mathbf{u}_4$ is turned on for the time duration $\Delta \tau_0$.

The equivalent machine circuit during $\mathbf{u}_4$ is shown in Fig. 16(b). The analysis of this switching state leads to a similar set of equations as (26 - 29), but with the sign of $U_d$ reversed. The measured phase voltages (30) at switching state $\mathbf{u}_4$ define the sample $u_{\sigma}^{(4)}$, where the superscript refers to the switching state $\mathbf{u}_4$. The difference between the two samples

$$p_\sigma(\delta_N) = u_{\sigma}^{(1)} - u_{\sigma}^{(4)} = 2U_d \frac{l_{\sigma a} + l_{\sigma c} - 2l_{\sigma b} \frac{l_{\sigma a} + l_{\sigma c} + l_{\sigma b}}{l_{\sigma a} + l_{\sigma b} + l_{\sigma c}} \quad (33)$$

is interpreted as the $a$-axis position signal. It depends only on total leakage inductances, since $U_d = \text{const}$, and the total leakage inductances vary with the rotor position. The value $p_\sigma(\delta_N)$ is immediately available when the additional switching state $\mathbf{u}_4$ has become active.

5.2 Polyphase position signals

The $a$-axis position signal defined in the previous paragraph can be measured whenever the inverter switching states $\mathbf{u}_1$, and subsequently $\mathbf{u}_4$, become active. More generally, one component of a polyphase position signal can be measured when any arbitrary first switching state $\mathbf{u}_{x1}$ occurs, which is followed by a subsequent second switching state $\mathbf{u}_{x2}$ that satisfies the condition $u_{x2} = -u_{x1}$. There exists a total of three independent pairs of switching states in a two-level inverter which satisfy the aforementioned condition.

Applying the procedure of Section 5.1 to the remaining pairs of inverter switching states yields the $b$-axis position signal

$$p_b(\delta_N) = u_{\sigma}^{(3)} - u_{\sigma}^{(6)} = 2U_d \frac{l_{\sigma b} + l_{\sigma c} - 2l_{\sigma a} \frac{l_{\sigma b} + l_{\sigma c} + l_{\sigma a}}{l_{\sigma a} + l_{\sigma b} + l_{\sigma c}} \quad (34)$$

when the switching states $\mathbf{u}_3$ and $\mathbf{u}_6$ are involved. It yields the $c$-axis position signal

$$p_c(\delta_N) = u_{\sigma}^{(5)} - u_{\sigma}^{(2)} = 2U_d \frac{l_{\sigma c} + l_{\sigma a} - 2l_{\sigma b} \frac{l_{\sigma c} + l_{\sigma a} + l_{\sigma b}}{l_{\sigma a} + l_{\sigma b} + l_{\sigma c}} \quad (35)$$

when switching states $\mathbf{u}_5$ and $\mathbf{u}_2$ are involved. The respective directions of the axes are marked in Fig. 15. The three signals $p_{\sigma}(\delta_N), p_b(\delta_N), p_c(\delta_N)$ form a symmetrical three-phase system whenever the number $N$ of
rotor slots is not a multiple of 3, which is true for almost every induction motor. This permits defining a complex position vector

\[ \mathbf{p}(\delta_N) = \frac{2}{3} \left( p_a(\delta_N) + a p_b(\delta_N) + a^2 p_c(\delta_N) \right) = p_\alpha + j p_\beta \]

(36)

where \( a = \exp(j2\pi/3) \). The important information contained in \( \mathbf{p}(\delta_N) \) is the angular rotor position \( \delta_N \) as referred to \( 1/N \) of a mechanical revolution, being expressed by the phase angle of \( \mathbf{p}(\delta_N) \), which is \( \delta_N \). The magnitude of \( \mathbf{p}(\delta_N) \) bears no significance.

The rotor position \( \delta \) within a full revolution is obtained by incrementing (or decrementing at reversed rotation) a modulo-\( N \) counter whenever a full cycle of \( \delta_N \) is completed. Hence the incremental rotor position within a full revolution is

\[ \delta = (2\pi C_\delta + \delta_N)/N \]

(37)

where \( C_\delta \) is the state of the counter. The equation illustrates the nature of this sensorless position measurement, which is absolute and of high resolution within one slot pitch, and incremental in that the angular displacements of the entire rotor slots are counted. It is quasi-instantaneous and hence can be obtained at high bandwidth.

6. EXPERIMENTAL RESULTS

6.1 Position measurement

The position identification method was tested using different induction motors. The following oscillograms were taken from a star-connected 7.5-kW motor of a machine tool spindle drive having \( 2p = 4 \) poles and \( N = 56 \) rotor slots.

The induction motor was fed by a PWM inverter. The phase voltages were measured at the motor terminals and, to ensure potential separation, transmitted to ground level as a high-frequency PWM modulated signal through small ferrite transformers.

Fig. 17 shows the recorded three-phase components \( p_a, p_b \) and \( p_c \) of the position signal at steady-state operation and no-load. The stator frequency is 0.1 Hz which corresponds to 3 rpm or 0.2 % of rated speed. Note that

![Fig. 17: Phase components \( p_a, p_b \) and \( p_c \) of the position signal measured at 0.1 Hz stator frequency](image)

the amplitudes of the position signals do not depend on speed, as seen in (33) - (35). This makes the measurement approach particularly suitable for the low speed range and for operation at standstill.

If the induction motor is operated at full excitation, the position signals exhibit an additional component of double fundamental frequency as seen in Fig. 18. Magnetic saturation leads to a periodic variation of the total leakage inductances. The disturbance is eliminated from the position signals by a lowpass filter operated in a \( 2\omega_1 \) coordinate system, similar to the filter shown in Fig. 4.

The components \( p_\alpha \) and \( p_\beta \) of the complex position vector (36) are shown in Fig. 19(a), and the near circular trajectory in the complex plane of the position vector \( \mathbf{p}(\delta_N) \) in Fig. 19(b). The high angular resolution and the smoothness of the signal permits high-accuracy position measurement, particularly because one full revolution of \( \mathbf{p}(\delta_N) \) represents only one \( N \)-th of a revolution of the motor shaft.

![Fig. 18: Phase components \( p_a, p_b \) and \( p_c \) of the position signal at rated load. The harmonic components of double fundamental frequency are due to magnetic saturation. The oscillogram was recorded at 1 Hz stator frequency](image)

![Fig. 19: Two-axes components \( p_\alpha \) and \( p_\beta \) of the complex position vector (a), and the resulting trajectory of \( \mathbf{p}(\delta_N) \), (b)](image)
Fig. 20 shows a position measurement in the steady-state at 0.1 Hz stator frequency. The oscillogram was recorded over one revolution of the motor shaft. It illustrates that the high frequency of the position signals enables accurate resolution of position measurement.

The position measurement is quasi-instantaneous since a pair of samples that permits the evaluation of one of the equations (33) - (35) is taken within a few microseconds. The position sampling frequency was set as 1 kHz. This ensures a high dynamic bandwidth of the position signal, thus enabling fast sensorless control of both speed and position.

6.2 Sensorless position and speed control

Smooth and accurate operation in the very-low speed range is not possible with state-of-the-art sensorless ac drives. The problem is solved by deriving a precise speed signal from the available position information. Fig. 21 shows the performance at speed control during speed reversal between ±10 rpm, or 6.7·10⁻³ rated speed. In this situation, the motor takes 20 seconds to perform one revolution of the motor shaft.

Sensorless position measurement has been implemented in a vector controlled drive system. Fig. 22 shows the characteristic signals for a positioning cycle executing periodic variations of the motor shaft angle of ±45°. The command of the angular displacement is doubled to ±90° in the oscillogram Fig. 23. A high transient torque of 120% nominal value is produced that maintains about the same displacement time as in Fig. 22. Magnetic saturation of the leakage paths causes the amplitudes of the position signals to temporarily reduce. This has no detrimental effect on the accuracy of the position measurement. Sustained operation at zero stator frequency and the dynamic performance at 120% overload is demonstrated in Fig. 24.

6.3 Performance of standard induction motors

Design details of the machine rotor determine the quality of the acquired position signals. The leakage inductance depends on the magnetic saturation in the main flux path since this flux also penetrates the rotor teeth. On the other hand, the load condition determines the rotor current which in turn contributes to the saturation of the rotor teeth. Both effects superimpose in a nonlinear relationship since the spatial directions of the two flux components differ. The resulting effect of magnetic saturation is a distortion of the position signals. The dominant frequency
is the second harmonic of the stator frequency as seen in Figs. 17, 18 and 25.

Adaptive filtering techniques are required to extract the position signals in the presence of second harmonic stator frequency components since these depend on the excitation of the machine, and on the load conditions.

The skew of the rotor bars tends to blur the measured position signals and to reduce their amplitudes. As seen from the stator, the rotors slot do not have unique position if the rotor is too much skewed. Fig. 26 shows the extreme of an acquired signal from a machine having a rotor skew of 2.5 times the rotor slot pitch. Such machine is not suited for position detection.

7. SUMMARY

Sensorless position control of induction motors is a promising new technology, though in the early state of development.

The saliency method uses a custom designed machine rotor to enable the estimation of the incremental rotor position. A high-frequency flux wave is injected into the stator as a test signal, from which a rotor position depending stator current component is generated. The position estimation consists of saliency models and compensators, and of a model of the mechanical subsystem of the drive, all incorporated as part of a phase-locked loop. Laboratory tests have have demonstrated the feasibility of position estimation at near zero stator current and torque, while closed loop position control has yet to be proved.

A different approach is based on the instantaneous measurement of the total leakage reactance of a standard induction motor. It requires the modification of the PWM switching signals, so as to subject the machine to defined transient conditions of very short durations. The position signal is derived from the measured stator voltages. A laboratory drive was operated as a high-resolution sensorless positioning system. It exhibits the dynamic performance of a vector controlled system, and is capable of smooth rotation at very low crawling speed. Extreme speed accuracy and good positioning accuracy are maintained throughout the operating range from no-load to full load. The method is applicable to most standard induction motors.

8. REFERENCES