Acquisition of Rotor Anisotropy Signals in Sensorless Position Control Systems

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Abstract – Sensorless position control of induction motors relies on the detection of rotor anisotropies. The repetitive transient excitation through the switched voltages of the PWM inverter provides the test signals to identify the spatial orientation of the anisotropies that indicate the rotor position. The position information is contained in the inverter output voltages. These are measured against the neutral point of the star connected stator winding. The signals are heavily corrupted by disturbances. These originate from the propagation of travelling waves on a long motor cable and from the influence of high-frequency common mode currents. The paper describes how a clean position signal is extracted in the presence of such noise.

Keywords: Induction motor, sensorless position control, rotor anisotropy, common mode currents, long motor cable, control at zero stator frequency

I. INTRODUCTION

Sensorless vector control of induction motors at zero stator frequency constitutes a problem of specific nature. As the electromagnetic field does not rotate in this operating point, rather maintaining a fixed position in space, the rotor induced voltages in the stator windings assume zero values. This makes it impossible to estimate the spatial orientation of the rotor field based on measured electrical quantities – voltages and currents – of the stator windings.

While the fundamental field model of the induction motor thus fails to provide full controllability at zero stator frequency, solutions to this problem can be found by making use of the anisotropic properties of the machine rotor. Squirrel cage rotors generally exhibit two kinds of anisotropies [1]. A first anisotropic property is caused by magnetic saturation. The spatial variations of the main field change the magnetic permeability of the rotor and the stator iron. The inductance values of the three stator phase windings therefore vary as a function of the displacement angle between the field axis and the respective phase axis. This effect permits estimating the spatial orientation of the main field. The field angle thus obtained is the key quantity for implementing a field oriented control system.

A second anisotropic property of the rotor is caused by the existence of \( N_f \) discrete conductors along the circumference of the cage rotor. Such structure creates two effects: First, the inductors, having low magnetic permeability while being embedded in the rotor iron of high magnetic permeability, create a periodic modulation along the rotor circumference of the spatial magnetic characteristics. Secondly, the discrete conductors themselves, being short circuited, exert an influence on the leakage inductances of the respective phase windings. The leakage inductance of one phase winding varies continuously, assuming a minimum and a maximum value as the rotor angle displaces by one rotor bar pitch. Since the number of rotor bars is generally not a multiple of three, the phase angles of the stator leakage inductance signals are displaced with respect to each other by \( 2\pi / 3 \). Measuring the total leakage inductances of the three stator phases therefore permits determining the absolute rotor position within one rotor bar pitch [2].

It is advantageous to measure the spatial orientations of the rotor anisotropies while subjecting the machine to a transient condition. This creates magnetic field components that propagate in space at velocities different from the velocity of the fundamental field, thus making the result independent of the prevailing load condition. Transient conditions are established by injecting frequency components other than the fundamental frequency, and generally higher, into the stator windings. The resulting transients can be exploited in two different ways: One class of methods evaluates the response of the machine to an injected periodic high-frequency signal, generally referred to as carrier signal [3, 4, 5]. An alternative approach utilizes the response to the stepped stator voltage waveforms that occur at pulsewidth modulated (PWM) control [2]. The latter method provides a high spatial resolution of the acquired angular position signal, and exhibits high dynamic bandwidth and a high signal-to-noise ratio. It is therefore given preference in the following.

The paper discusses first how the rotor bar position angle is determined from the measured voltages at the inverter output terminals. It shows further that travelling wave phenomena and high-frequency common mode currents tend to disturb the measured signals, especially when a longer cable exists between the inverter and the drive motor. Methods to eliminate the disturbances are presented, permitting to extract clean
and accurate rotor anisotropy signals from a PWM controlled motor drive. These signals define the field angle, or the rotor bar position angle, even at zero stator frequency or zero speed. Using these signals for sensorless position control of an induction motor is the subject of a previous publication [6].

II. ACQUISITION OF THE ROTOR ANISOTROPY SIGNAL

The anisotropic properties of a squirrel cage rotor are reflected in variations of the total leakage inductances \( l_\sigma \) of the stator phase windings. Fig. 1 shows how the respective phase components change when the cage rotor displaces by a spatial angle corresponding to one pole pair, i.e. it displaces over a full electrical revolution. With the machine operated at no load and full magnetic excitation, the saturation anisotropy accounts for the two respective minimum and maximum values per electrical revolution. Superimposed to this is the effect of the rotor cage anisotropy, characterized by \( N_r \) oscillations per mechanical revolution, where \( N_r \) is the number of rotor bars and \( p \) is the number of pole pairs. It is seen in addition that the amplitude of the cage anisotropy component depends on the saturation level. Note that the two anisotropies rotate at different angular velocities when the machine is loaded.

A signal reflecting the anisotropic characteristics of a squirrel cage rotor can be acquired following a transient excitation of the machine. Such excitation is perpetually generated by the switching of the inverter. The respective inverter states are characterized by the eight switching state vectors shown in the right of Fig. 3. Let a particular inverter switching state be \( u_1 \). The notation \( u_1 \) indicates that the phase potentials \( u_a = U_d/2 \) and \( u_b = u_c = -U_d/2 \), where \( U_d \) is the dc link voltage.

This situation is represented by the equivalent circuit Fig. 2. The following equations hold

\[
\begin{align*}
  u_1 &= l_0 \frac{di_1}{d\tau} + u_i \\
  \text{(1)}
\end{align*}
\]

where \( l_0 \) and \( u_i \) are the space vectors of the stator current and the rotor induced voltage, respectively, and

\[
I_\sigma = \begin{bmatrix} l_{\sigma a} \\ l_{\sigma b} \\ l_{\sigma c} \end{bmatrix}
\]

is the leakage inductance tensor, the three-phase components of which are the respective phase values \( l_{\sigma a}, l_{\sigma b}, l_{\sigma c} \) of the total leakage inductance. These inductances would equal each other with an isotropic rotor, \( l_{\sigma a} = l_{\sigma b} = l_{\sigma c} \). In such case, the zero sequence component, expressed by the sum of the three phase voltages, amounts to zero,

\[
  u_a + u_b + u_c = 0,
\]

since the induced voltages \( u_{ia}, u_{ib}, u_{ic} \), being almost sinusoidal, do not exhibit a zero sequence component,

\[
  u_{ia} + u_{ib} + u_{ic} = 0,
\]

and consequently

\[
  \frac{di_a}{d\tau} + \frac{di_b}{d\tau} + \frac{di_c}{d\tau} = 0.
\]

An anisotropic rotor introduces an unbalance to the phase values of the total leakage inductances as demonstrated in the simulated curves Fig. 1. It is assumed that the unbalances vary sinusoidally when the anisotropy displaces in space as the rotor and its magnetic field rotate. The transient voltage components \( l_{\sigma a}di_a/d\tau, l_{\sigma b}di_b/d\tau, l_{\sigma c}di_c/d\tau \) of the first term in (1) then vary as the total leakage inductance values vary. As a consequence, the derivative \( di_a/d\tau \) in (1) assumes a different direction than the driving voltage vector \( u_1 - u_1 \).

To illustrate the effect, it is assumed that the anisotropic, e.g. saturated rotor in Fig. 3 does not move: \( \omega_s = 0 \) and hence \( u_1 = 0 \) in (1). The leakage inductance tensor \( I_\sigma \) makes the derivative \( di_a/d\tau \) point in a different direction than the exciting voltage vector \( u_1 \). The physical explanation is that the low leakage inductance in the \( d \)-axis creates a greater current derivative component than the high leakage inductance in the \( q \)-axis.

One way of identifying the spatial orientation of the anisotropy is to determine the transient component \( di_a/d\tau = u_1 - u_1 \)
of the stator current derivative \( \frac{dl_i}{dt} \). Its locus moves on a circle as shown in Fig. 3, having an angular displacement \( 2\delta \) where \( \delta \) is the spatial displacement angle of the anisotropy.

A simpler method consists in analyzing only the scalar component \( \frac{dl_i}{dt} \) in the direction of the exciting voltage vector, which is \( u_1 \) in this example. This component induces the transient voltage \( l_{ia} \frac{dl_i}{dt} \) in stator phase \( a \). Also the components \( l_{ib} \frac{dl_i}{dt} \) and \( l_{ic} \frac{dl_i}{dt} \) of the other two phases are affected by the respective leakage inductance values \( l_{ib} \) and \( l_{ic} \), which are as well anisotropy dependent.

These dependences have an effect on the potential of the neutral point \( N \) in Fig. 2, which makes the zero sequence voltage assume nonzero values. Equation (3) converts to

\[
\sigma = u_a + u_b + u_c
\]

which expression defines an anisotropy related signal. This signal is acquired by measuring and adding the phase voltages \( u_a, u_b, \) and \( u_c \).

To investigate the properties of the anisotropy signal \( \sigma \) (7), equations (1) through (4), (6) and (7) are solved, which yields

\[
\sigma^{(1)} = u_d \frac{l_{ia}}{l_{ia} + l_{ib} + l_{ic}} - 2 \frac{l_{ib} l_{ic}}{l_{ia} + l_{ib} + l_{ic}} + u_{\sigma 1}
\]

where the term

\[
u_{\sigma 1} = 3 - \frac{l_{ia} l_{ib} l_{ia}}{l_{ia} + l_{ib} + l_{ic}}
\]

represents the contribution of the rotor induced voltages \( u_{ia}, u_{ib}, \) and \( u_{ic} \). These voltages are small at low speed and hence can be neglected. The influence of \( \sigma_{1} \) at higher speed is discussed in [6].

The superscript in the notation \( \sigma^{(1)} \) in (8) indicates that the anisotropy signal refers to a transient excitation by the switching state vector \( u_1 \).

It is obvious from the inspection of Fig. 3 that transient excitations by different switching state vectors produce different values of the anisotropy signals \( \sigma^{(2)}, \sigma^{(3)}, \ldots \), however, \( \sigma^{(4)} = -\sigma^{(1)}, \sigma^{(5)} = -\sigma^{(2)}, \) and \( \sigma^{(6)} = -\sigma^{(3)} \).

The anisotropy signal \( \sigma^{(1)} \) is displayed in Fig. 4. The signal exhibits very favorable characteristics:

- The amplitude of the rotor cage related anisotropy component, which is the high-frequency oscillation, is much more uniform than the corresponding components of the phase leakage inductances, Fig. 1.
- Other than the phase leakage inductance curves \( l_{ia}, l_{ib}, \) and \( l_{ic} \) in Fig. 1, the anisotropy signal \( \sigma^{(1)} \) does not exhibit a dc offset.

The formal reason for these favorable properties is in the nonlinear mapping of the phase leakage inductances on the anisotropy signal expressed by (8). The summing of the phase voltages in (7) further eliminates all nonsignificant large fundamental components, and the dc offset as well. The small changes in the curve \( l_{ia} \) therefore transform a balanced ac signal having a remarkable amplitude of more than 50 V. This ensures a very high signal-to-noise ratio.

III. SIGNAL DISTURBANCES

3.1 Parasitic effects

The discussion in the preceding paragraph are based on idealized conditions; the results were obtained by simulations. The conditions in a real inverter drive system are quite different. The common mode voltages generated by the switching of the inverter produce oscillating currents of high frequency that close...
through the stray capacitances of the motor windings versus ground, from where they reenter the drive system through the feeding utility transformer, the line-side diode bridge converter and the dc-link circuit [7].

In addition, travelling waves propagate along the three-phase cable that connects the inverter to the drive motor [8]. They are partially reflected from the motor terminals back to the inverter; another portion penetrates into the stator windings, thus creating oscillations of the common mode voltages. With these disturbances existing, it is almost impossible to measure the anisotropy signal (7) with the required accuracy. Steps must be taken, therefore, to reduce the influence of the parasitic oscillations.

3.2 Common mode currents

An analysis of the transient processes caused by the common mode voltage of the inverter is based on the equivalent circuit Fig. 5. The motor is represented here by the leakage inductances of its three phases. The distributed capacitances of the inverter, the motor cable and the motor windings against ground are approximated by the lumped capacitor $C_g$.

The balanced fundamental three-phase voltage system that feeds the drive motor represents one component of the pulsewidth modulated waveforms. The respective potentials of the three inverter terminals referred to the center point $m$ of the dc link circuit are restricted to either $+U_d/2$ or $-U_d/2$. The sum of the three phase potentials is always nonzero, although time-variable. Its common mode component against ground is

$$u_{abc,g} = \frac{1}{3}(u_a + u_b + u_c)$$

which can be also written as

$$u_{abc,g} = \frac{1}{3}(u_{am} + u_{bm} + u_{cm}) + u_{mg}$$

where $u_{am}$, $u_{bm}$, and $u_{cm}$ are the voltages between the respective motor terminals and the dc link mid point, and $u_{mg}$ is the voltage of the mid point against ground.

Table 1 shows the values of the common mode voltage $u_{abc,g}$ as a function of the eight possible switching state vectors, where $u_0 = u_6 = (- -)$ and $u_7 = (+++)$. The Table indicates that $u_{abc,g}$ does not only depend on the respective switching state vectors, but also on $u_{mg}$. The voltage $u_{mg}$ itself is variable which will be analyzed in the following.

3.2.1 Zero switching state vectors.

The line-side rectifier in Fig. 5 operates predominantly in the discontinuous conduction mode. Owing to the dc link capacitor $C_d$, the dc link voltage $U_d$ is nearly constant and mostly higher than the phase-to-phase voltages of the mains. The line currents then tend to decrease to zero after each short interval in which the capacitor gets charged.

It is assumed first that the inverter maintains one of its zero states $u_0$ or $u_7$. Considering the particular charging interval in which the diodes D1 and D6 conduct, the following equation holds

$$u_{mg} = \frac{1}{2}U_d + l_i \frac{dI}{dt} + u_{bl} = \frac{1}{2} u_{abl} + u_{bL} = -\frac{1}{2} u_{cL}.$$  \hspace{1cm} (12)

where $l_i$ is the line inductance per phase and $i_j$ is the charging current in the loop D1 $- C_d$ $-$ D6. Hence, during the charging interval $u_{mg}$ equals $-u_{cL}/2$. The voltage of the capacitor $C_g$ is then $-u_{cL}/2 - U_d/2$ when the inverter is in switching state $u_0$; it equals $-u_{cL}/2 + U_d/2$ when the inverter is in switching state $u_7$. When the diodes return to a blocking state after the short charging interval, the

<table>
<thead>
<tr>
<th>switching state vector</th>
<th>$u_{abc,g}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u_0$</td>
<td>$u_{mg} - U_d/2$</td>
</tr>
<tr>
<td>$u_1$, $u_3$, $u_5$</td>
<td>$u_{mg} - U_d/6$</td>
</tr>
<tr>
<td>$u_2$, $u_4$, $u_6$</td>
<td>$u_{mg} + U_d/6$</td>
</tr>
<tr>
<td>$u_7$</td>
<td>$u_{mg} + U_d/2$</td>
</tr>
</tbody>
</table>

Table 1. Common mode voltage $u_{abc,g}$ at different switching states.
inverter gets disconnected from the feeding line, and the voltage of the capacitor $C_g$ remains constant. Also $u_{mg}$ remains constant, keeping the value it had when the diodes disconnected. Fig 6 shows the respective simulated and measured waveforms.

### 3.2.2 Active switching state vectors.

Additional common mode voltages are generated when the inverter operates in the pulsewidth modulated mode. Table 1 shows that the common mode voltage $u_{abc, g}$ in addition to $u_{mg}$, steps up by $U_d/3$ increments during one PWM half-cycle, e.g. $u_0 - u_1 - u_2 - u_7$, stepping down by $U_d/3$ decrements in the subsequent half-cycle $u_7 - u_2 - u_1 - u_0$. It is assumed first that the line-side bridge rectifier is in a state of conduction. The stray capacitances of the system, represented by the lumped capacitor $C_g$ in Fig. 5, are then periodically charged or discharged, following the changes of the common mode voltage at each commutation of the inverter. The charging loop closes through ground, the utility transformer, and the diode bridge. Owing to the inductances of this circuit, the charging current is superimposed by damped high-frequency oscillations.

The situation is different when the diodes of the line-side bridge rectifier are in the blocking phase of the discontinuous conduction mode. Considering the same phase interval of the line voltages as in Section 3.2.1, it is the diodes D1 and D6 that are exposed to the least blocking voltage from the line according to

$$u_{abL} - U_d = u_{D1} + u_{D6}$$  \hspace{1cm} (13)

where $u_{abL} < U_d$ and hence $u_{D21}, u_{D2} < 0$. Taking the pulse-width modulated operation of the inverter into account, positive step changes of the common mode voltage $u_{abc, g}$ occur in the course of one of the switching patterns that are listed as a downward sequence in Table 1. Fig. 5 shows that such step changes will forward-bias diode D1. Following each step, a common current flows through D1, first charging its reverse blocking capacitance and then transferring D1 to the conduction mode. Although originally nonconducting, the diode D1 connects the dc link to the mains with the effect that the voltage $u_{mg}$ gets limited to $u_a - U_d/2$.

Conversely, negative step changes of the common mode voltage $u_{abc, g}$ occur in the course of any upward sequence of the switching patterns as per Table 1. The resulting common mode current has then initially the opposite polarity, thus forcing D6 into conduction which limits the voltage $u_{mg}$ to $u_b + U_d/2$.

Similar conditions exist when the phase angle of the line voltage advances and the least blocking voltage occurs across other diode pairs. As a consequence, the waveform of $u_{mg}$ exhibits the characteristic pattern shown in Fig. 7. The envelopes of the high-frequency oscillations are formed by the respective utility phase voltages.

Whenever the common mode voltage changes in a step fashion, the common mode currents, after an initial transient, develop a fast ringing component as the distributed capacitances of the machine winding, represented by $C_g$, and the stray inductances resonate. The respective power diodes of the bridge rectifier, having once initiated conduction, cannot react fast enough when the ringing currents reverse; they tend to maintain their internal carrier distribution and hence remain their original state of conduction.

An oscillogram of the common mode current is shown in Fig. 8(a). Its frequency is about 2 MHz. This high value indicates that the leakage inductances of the motor and the mains are not fully effective; they are partially bypassed as the steep current gradients find a parallel path through the winding ca-

![Fig. 8. Measured waveforms of the common mode current following an inverter commutation](image-url)

![Fig. 9. Propagation of travelling waves on the motor cable and in the stator windings](image-url)

![Fig. 10. Equivalent circuit per unit length $dx$ of one stator phase winding](image-url)
The inverter is generally mounted in a control cubicle, while the drive motor, forming part the plant, is situated in a different location. The connecting three-phase cable can be of considerable length. It can be modelled as a distributed parameter system [7]. The cable is excited by the voltage steps of the inverter. These propagate along the cable length as individual forward travelling waves per phase, 
\[ u_t = u_{inv}, \quad i_t = u_t / Z_c \]  
where the subscript \( t \) refers to a forward travelling wave and \( Z_c \) is the natural impedance of the cable. \( i_t \) is the current associated to the forward wave. The waves travel at a finite velocity of about 160 m/s.

Their propagation is schematically illustrated in Fig. 9, assuming that a deenergized motor cable is excited by the switching state vector \( u_1 (+-+) \). At time instant \( t_1 \), the forward waves have reached the marked positions in Fig. 9.

Since the stator winding impedance \( Z_s \) is higher than the natural impedance \( Z_c \) of the motor cable, the reflection factor at the motor terminals is
\[ \Gamma_s = \frac{Z_s - Z_c}{Z_s + Z_c} < 0. \]  
where \( Z_s \) is the terminal impedance of the stator winding.

The transient conditions produce travelling waves in the motor windings as well. Therefore, as the forward waves reach the motor terminals, they bifurcate to initiate two new travelling waves per phase: Assuming in a first step that \( \Gamma_s \) is frequency independent, the motor cable develops the reverse travelling waves
\[ u_t = \Gamma_s u_t, \quad i_t = -u_t / Z_c, \]  
while a set of forward waves starts penetrating into the stator windings. The respective voltages and currents at the motor terminals are defined by
\[ u_{fs} = u_t + u_r, \quad i_{fs} = u_{fs} / Z_s \]

The propagation of forward waves in the motor winding is marked in Fig. 9 as corresponding to time instant \( t_2 \). The phase windings of the motor behave as a distributed parameter system, similar to that of an electric line. There is a difference, though, since additional capacitances exist between the individual turns of a multi-turn winding. The equivalent circuit per unit length \( dx \), Fig. 10, models the winding capacitances by the additional element \( G'/dx \) [8].

An incident wave appearing at the motor terminals is therefore split into two portions: An undercritical and an overcritical component [9]. It is apparent from Fig. 10 that frequency components above a critical frequency are predominantly short-circuited by the two distributed capacitances \( G'/dx \) and \( C'dx \). The do not reach further into the winding than the first few turns. Hence the reflection factor for these components is \( \Gamma_s \approx -1 \), which indicates that higher frequency waves are fully reflected at the motor terminals. They travel back to the inverter.

Only the lower frequency components penetrate into the stator windings as described by (17). The intermediate frequency components in the neighborhood of the critical frequency
\[ \omega_c = \frac{1}{l_s \sqrt{L'G'}}, \]  
where \( l_s \) is the conductor length of one stator phase winding, are partially reflected to return on the motor cable, and partially transmitted to penetrate into the stator windings. The reflection factor \( \Gamma_s \) in (16) is therefore frequency dependent. This influences upon the shape of the travelling waves that propagate in the stator windings. Their step front converts into a wedge-like shape as shown in the right of Fig. 9. The front length [9] of the modified wave is
\[ l_f = \pi l_s \sqrt{\frac{G}{L}}. \]
The reverse wave on the motor cable is again reflected at the inverter terminals, where $\Gamma_{\text{inv}} < 0$. The reflections between the motor and inverter terminals continue while the wave energy reduces, being partly absorbed in the stator windings and partial consumed by losses. Similar processes take place in the stator windings. The modified forward waves are reflected at the starpoint $N$, which is short-circuited and hence $\Gamma_N = -1$. The reverse waves are then again reflected at the motor terminals, which process continues until a steady-state is reached.

Since the distributed capacitances and inductances of the motor windings are much higher than those of the motor cable, the propagation velocity and hence the frequency of the periodic reflections in the motor is lower than that of the cable. Two different frequencies are therefore superimposed on the stator phase voltage, Fig. 11(a). They are caused by travelling delay effects. The motor cable generates the high-frequency oscillations, while the stator windings produce the low frequency component.

IV. MEASUREMENT OF ROTOR ANISOTROPIES

To avoid running a separate measurement cable from the motor terminals to the inverter control unit, the anisotropy samples are sampled at the inverter terminals itself, Fig. 12. The anisotropy signal (7) then becomes

$$u_\sigma = u_\sigma N + u_\sigma N + u_{cN}.$$  \hspace{1cm} (20)

A connection to the motor starpoint $N$ is still needed for signal acquisition. The approach requires the following additional measures:

- Reducing the influence of high-frequency common mode currents.
- Reducing the influence of signal distortions caused by signal reflections on the motor cable.
- Introducing a delay interval between a transient excitation by the inverter and the signal sampling instant to account for the signal delay on the motor cable and in the stator windings.

4.1 Common mode currents

The anisotropy signal is measured following the transient excitations of the system during changes of the inverter switching state. According to Table 1, such changes are accompanied by the changes

$$\Delta u_{abc,g} = \pm \frac{1}{3} U_d + u_{mg}$$  \hspace{1cm} (21)

of the common mode voltage $u_{abc,g}$, where $\pm U_d/3$ are single step functions while $u_{mg}$ is the high-frequency voltage shown in Fig. 7.

While the diode bridge rectifier in Fig. 5 operates in the discontinuous conduction mode, the changes of the common mode voltage (21) force one particular rectifier diode during each 60°-degree interval of the line voltages into conduction. The effect was explained in Section 3.2.2. It leads to a 300-Hz amplitude modulation of the voltage component $u_{mg}$ in (21) as seen in Fig. 7. The resulting common mode current superimposes a clear 300-Hz component on the anisotropy signal as shown in Fig. 13.

The circuit arrangement in Fig. 12 eliminates this disturbance. The two capacitors $C_{p1}$ and $C_{p2}$ deviate the oscillating common mode current from the line-side rectifier. The current is then impeded from forcing the pair of near-conducting rectifier diodes (D1 and D6 in Section 3.2.2) into alternating conduction. The voltage $u_{mg}$ remains almost constant as a consequence. In addition, a set of coupled inductors $L_c$ between the inverter output and the motor cable constitutes a high impedance for the common mode currents. These reduce in amplitude as shown in Fig. 8(b).

4.2 Reflections of travelling waves

A better impedance matching at the motor side of the cable can be achieved by a three-phase $RC$ filter as shown in Fig. 12. Such filter adjusts the reflection factor $\Gamma_s$ in (15) close to zero for higher frequencies if $R_f = Z_0$ is chosen. The value of the filter capacitor $C_f$, on the one hand, determines the frequency range for which impedance matching is achieved, and the filter losses and the other. A good compromise is $C_f = (3$
4.3 Delayed sampling

The propagation of the switching transients in the stator windings occurs at relatively low velocity. Travelling wave reflections then produce the low-frequency oscillations in the phase-to-neutral voltages that are seen in Fig. 11. Although it appears desirable searching for means to eliminate the oscillations, it must be realized that the wave propagation process and the expansion of the transient leakage fields are physically inseparable. The fact requires the anisotropy signal be sampled before the transient leakage fields of the machine decay [10]. However, the high-frequency components of the phase-to-neutral voltages introduce an error when the samples are taken too early.

An even more stringent reason for taking the measurement samples with minimum delay after the transient excitation is given by the PWM algorithm. The active switching state vectors, used for transient excitation prior to signal acquisition, occur at a very short duration when the stator frequency, and hence the fundamental stator voltages, are low. The on-times of the active vectors may be even lower than the minimum commutation interval of the inverter, which is in the microsecond order. It appears impossible, even with the improvement achieved by additional filters in the power circuit, to take accurate measurement samples after such short time delay.

The solution consists in taking the samples while the low-frequency oscillations still persist, but having assumed a defined form with the high-frequency oscillations already disappeared. The approach is illustrated with reference to the oscillograms of the anisotropy signal $u_\sigma$, Fig. 15. The $RC$ filter at the motor terminals reduces much of the high-frequency disturbance caused by the motor cable. The signal is sampled at $t_s + T_d$, at which time instant the high-frequency oscillations have died out. $T_d = 3.4 \mu s$ was chosen in our setup, which means that the sample is taken at the first overshoot of the low-frequency oscillation. This sample is termed $u_\sigma'$. A correction is then made to estimate its steady-state value $u_\sigma$.

The following modification is introduced for this purpose

$$u_\sigma = \frac{u_\sigma'}{1 + k_s}$$  \hspace{1cm} (22)

where $k_s$ is the overshoot percentage. Note that the amplitude of $u_\sigma$ is small as compared with the amplitudes of the high-frequency disturbances.

The corrected anisotropy signals are displayed in Fig. 16. The low-frequency component is caused by magnetic saturation. Its separation from the high-frequency signals is described in [6]. The latter signals are induced by the rotor cage anisotropy. The waveform $u_{\sigma a}$ is composed of either $u_1^{(1)}$ at transient excitation by $u_1$ as per (8), or $-u_1^{(1)}$ at transient excitation by $u_4 = -u_1$, likewise is $u_{\sigma b}$ composed of either $u_\sigma^{(5)}$ or $-u_\sigma^{(5)}$, and $u_{\sigma c}$ of either $u_\sigma^{(1)}$ or $-u_\sigma^{(5)}$. Accordingly, $u_{\sigma a}$ relates to a transient excitation in phase axis $a$, $u_{\sigma b}$ in axis $b$, and $u_{\sigma c}$ in axis $c$.

The highlighted section in Fig. 16 indicates that the rotor anisotropy signals $u_{\sigma a}$, $u_{\sigma b}$, $u_{\sigma c}$ form a balanced three-phase system that defines a unique phase angle, which is the rotor bar position angle. The signals are sampled at the high repeti-
tion rate of 2 kHz which ensures a high dynamic bandwidth of the rotor position measurement. They are continuous and smooth which guarantees a high spatial resolution.

4.4 Motor Data
The induction motor used for the experiments has the following data: $P_R = 4.5 \text{ kW}$, $U_R = 360 \text{ V}$, 50 Hz, $I_R = 9.26 \text{ A}$, cos $\phi = 0.86$, 1730 rpm, $2p = 4$, $n_{	ext{TP}} = 22$, $R_s = 1.026 \text{ } \Omega$, $R_t = 0.639 \text{ } \Omega$, $L_s = 4.48 \text{ mH}$, $L_t = 6.29 \text{ mH}$, $L_m = 140 \text{ mH}$, no skew.

**SUMMARY**

The squirrel cage rotor of an induction motor is an anisotropic structure. Its spatial orientation influences on the respective phase values of the total leakage inductances of the stator winding. The switched waveforms of the pulsewidth stator voltages subject the machine to repetitive transient excitations. These serve to extract a signal that represents the absolute angular position of the rotor bars. The method requires measuring the phase-to-neutral voltages at the inverter terminals. Disturbances originating from high-frequency common mode currents and travelling wave phenomena on long motor cables are eliminated by appropriate filter and signal sampling techniques.

**REFERENCES**