Sensorless Acquisition of the Rotor Position Angle of Induction Motors with Arbitrary Stator Windings

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Abstract — The anisotropy of a cage rotor is utilized to determine the angular position of the rotor in an induction machine. The switching transients generated by a pulsewidth controlled inverter serve as test signals. The response of the three inverter terminal currents is exploited to derive a quasi-instantaneous rotor position signal. The position is sensed at the inverter through the 3-phase motor cable by measuring the current derivatives. The method does not require additional wire connections. It is applicable to induction motors having the stator windings connected either in wye or in delta.

The results are supported by measurements from an experimental setup.

Keywords: Induction motor, rotor anisotropy, sensorless position control, delta connected stator windings

I. INTRODUCTION

The cage rotor of induction machines is magnetically anisotropic. This is owed to the presence of its discrete rotor bars, being embedded in the respective slots of the rotor iron. The rotor anisotropies make the high-frequency impedance of the polyphase stator winding vary as a function of the rotor position angle. A transient excitation of the machine, superimposed on the stator voltages in any given operating condition of the machine, therefore produces a response of the stator currents that depends on the rotor position angle. A response signal can be extracted and processed in a suitable manner so as to derive a rotor position signal. Such signal is independent of the mechanical speed of the rotor; it can be obtained without compromise at very low speed and at zero speed [1]. It is well suited as a feedback signal for closed loop control of rotor position, or speed. Applications of interest are sensorless control of the angular rotor position, or, alternatively, sensorless speed control to promote stable operation in the low speed range and at zero speed [2]. Such performance attributes are difficult to obtain with more traditional sensorless techniques based on the fundamental model of the machine [3].

Transient excitation can be achieved by a high-frequency stator voltage component being added to the ideally sinusoidal stator voltage waveforms [4]. Also the switched PWM waveforms represent a transient excitation. Each transient has a wide-band harmonic content and repeats at the inverter switching frequency. It generates much larger amplitudes than an added voltage component and hence can be favorably exploited to acquire a rotor position signal [5]. More than that, the method does not require an additional voltage margin; it produces a high-bandwidth, high-accuracy position signal of high signal-to-noise ratio, well suitable both for sensorless position and sensorless speed control [2].

The response to the sudden changes of the switching state vector in a PWM controlled drive induces unbalanced dynamic voltages across the leakage inductances of three stator phases. The dynamic voltages create a zero-sequence component in the stator voltages which is extracted by adding the three phase voltages. A simple algorithm then serves to define a position signal [2]. The method requires a star-connected stator winding that permits measuring the phase voltages. For this purpose, an additional wire is required in the motor cable [5]. It makes the star point potential available at the feeding inverter unit.

The zero-sequence voltage is inherently zero in a delta-connected machine. The dynamic voltages then create a zero-sequence current which can be measured using a single probe through which the three phase currents are lead. Six wires must be brought out from the motor terminal box, and a separate cable is needed to make the measured and amplified signal available at the inverter unit [6].

Star-connected windings and additional wires, or separate measuring boxes having separate connections to the inverter unit constitute restrictions and additional installations that are not much appreciated. To overcome this problem, the paper proposes a solution that does not require additional installations and cables. Instead, the motor is fed from the inverter through a regular three-phase cable through which the rotor
position angle is sensed at the feeding inverter unit.

II. ANISOTROPY MODEL

The anisotropy of the rotor introduces an unbalance in the phase values of the stator leakage inductances. The resulting inductance values are described in good approximation [2] by

\begin{align}
    l_{aa} &= l_{\alpha 0} + l_\Delta \cos(n_{rp}\vartheta) \\
    l_{bb} &= l_{\alpha 0} + l_\Delta \cos\left(n_{rp}\left(\vartheta - \frac{2\pi}{3}\right)\right) \\
    l_{cc} &= l_{\alpha 0} + l_\Delta \cos\left(n_{rp}\left(\vartheta - \frac{4\pi}{3}\right)\right)
\end{align}

were \(l_{\alpha 0}\) is the average inductance, \(l_\Delta\) is its variation caused by the rotor anisotropy, \(n_{rp}\) is the integer number of rotor bars per pole pair, and \(\vartheta\) is the rotor position angle.

The assumption of a sinusoidal variation in (1) is confirmed by measured waveforms, e. g. Fig. 10. It should be noted, though, that the behavior of the machine at high frequency excitation is a highly complex phenomenon, and very difficult to model. In addition to the discrete winding model in [2], attempts have been made to consider the variable permeance of the magnetic circuit at different numbers of stator and rotor teeth as the basis of a model approach [7]. Such models could be valid in cases where \(n_{rp}\) is not an integer. The modelling problem gets further complicated by skin effect since the penetration depth into the rotor iron is only fraction of a millimeter. Machines with closed rotor slots are therefore not suited for position measurement. The same is true for machines having a skewed rotor if the skew equals the pitch of the rotor teeth.

The unbalanced leakage inductances force an unbalance on the stator current derivatives whenever a change of switching state vector occurs. To consider an example, it is assumed that the switching state vector \(u_1(+--\) is turned on. This establishes an inverter configuration as shown in the left of Fig. 1.

A. Stator winding in wye-connection

The case of a wye-connected stator winding is considered first. Fig. 2(a) shows the stator winding topology with the inverter in the active switching state \(u_1\). The following equations hold

\begin{align}
    \frac{d}{d\tau}i_a(u_1) &= \frac{u_d}{3l_{\alpha 0}}\left(1 - \left(\frac{l_\Delta}{27l_{\alpha 0}}\right)^2\right)\left(2 - \frac{l_\Delta}{l_{\alpha 0}}\cos(n_{rp}\vartheta)\right) \\
    \frac{d}{d\tau}i_b(u_1) &= \frac{u_d}{3l_{\alpha 0}}\left(1 - \left(\frac{l_\Delta}{27l_{\alpha 0}}\right)^2\right)\left(1 + \frac{l_\Delta}{l_{\alpha 0}}\cos\left(n_{rp}\left(\vartheta - \frac{2\pi}{3}\right)\right)\right) \\
    \frac{d}{d\tau}i_c(u_1) &= \frac{u_d}{3l_{\alpha 0}}\left(1 - \left(\frac{l_\Delta}{27l_{\alpha 0}}\right)^2\right)\left(1 + \frac{l_\Delta}{l_{\alpha 0}}\cos\left(n_{rp}\left(\vartheta - \frac{4\pi}{3}\right)\right)\right)
\end{align}

Restricting the analysis to operation at very low speed, the influence of the back-emf need not be considered in [2].

Equation (1) and (2), after some tedious calculations, are converted to

\begin{align}
    \frac{d}{d\tau}i_a(u_1) &= \frac{u_d}{3l_{\alpha 0}}\left(1 - \left(\frac{l_\Delta}{27l_{\alpha 0}}\right)^2\right)\left(2 - \frac{l_\Delta}{l_{\alpha 0}}\cos(n_{rp}\vartheta)\right) \\
    \frac{d}{d\tau}i_b(u_1) &= \frac{-u_d}{3l_{\alpha 0}}\left(1 - \left(\frac{l_\Delta}{27l_{\alpha 0}}\right)^2\right)\left(1 + \frac{l_\Delta}{l_{\alpha 0}}\cos\left(n_{rp}\left(\vartheta - \frac{2\pi}{3}\right)\right)\right) \\
    \frac{d}{d\tau}i_c(u_1) &= \frac{-u_d}{3l_{\alpha 0}}\left(1 - \left(\frac{l_\Delta}{27l_{\alpha 0}}\right)^2\right)\left(1 + \frac{l_\Delta}{l_{\alpha 0}}\cos\left(n_{rp}\left(\vartheta - \frac{4\pi}{3}\right)\right)\right)
\end{align}

The structure of these equations suggests defining a rotor bar position vector in the following form

\begin{equation}
    \vec{\psi}_r = \frac{2}{3}l_\Delta \cos(n_{rp}\vartheta) + a\cos\left(n_{rp}\left(\vartheta - \frac{2\pi}{3}\right)\right) + a^2\cos\left(n_{rp}\left(\vartheta - \frac{4\pi}{3}\right)\right)
\end{equation}

or
The phase components of the rotor bar position vector as functions of the stator current derivatives following the turn-on of the active switching state vectors $u_1$ through $u_6$

$$p_r = \frac{2}{3} \left( p_a + a p_b + a^2 p_c \right) = p_a + j p_b = p \cdot e^{j n p \theta}$$

The vector $p_r$, as seen from its argument $n p \theta$, indicates the angular position of the rotor within one rotor bar pitch. While the rotor displaces for one rotor bar pitch, the vector $p_r$ changes its argument by $2 \pi$.

The phase components $p_a, p_b$ and $p_c$ of the rotor bar position vector, as defined in (4) and (5), are introduced in (3) to obtain

$$p_a = -\frac{3 l_{\sigma 0} \left(1 - (l_\Delta/2 l_{\sigma 0})^2 \right)}{u_d} \frac{d i_a(u_1)}{d \tau} + 2$$  \hspace{1cm} (6a)$$

$$p_b = -\frac{3 l_{\sigma 0} \left(1 - (l_\Delta/2 l_{\sigma 0})^2 \right)}{u_d} \frac{d i_b(u_1)}{d \tau} - 1$$  \hspace{1cm} (6b)$$

$$p_c = -\frac{3 l_{\sigma 0} \left(1 - (l_\Delta/2 l_{\sigma 0})^2 \right)}{u_d} \frac{d i_c(u_1)}{d \tau} - 1$$  \hspace{1cm} (6c)$$

For a transient excitation by one of the other active switching states, $u_1$ through $u_6$, the respective components of the rotor bar position vector are represented by similar equations. They are listed in Table 1. The abbreviation

$$g = \frac{3 l_{\sigma 0} \left(1 - (l_\Delta/2 l_{\sigma 0})^2 \right)}{u_d}$$

is introduced to characterize a common factor in these equations.

B. Stator winding in delta-connection

The definitions in Fig. 2(b) are used to consider the case of a stator winding in wye-connection. Note that the line currents are referred to as $i_{a'}, i_{b'}, i_{c'}$, while the phase currents are $i_a, i_b, i_c$ in Fig. 2(a), and $i_{a'}, i_{b'}, i_{c'}$ in Fig. 2(b). This notation is convenient since only the line currents are used in the analyses. These are the currents that are measured at the inverter terminals to acquire a rotor position signal. Fig. 1 illustrates the approach.

It is again assumed that the inverter is in switching state $u_1$. The following equations are established for delta-connected windings:

$$\frac{d i_a(u_1)}{d \tau} = \frac{u_d}{l_{\sigma 0}} \frac{l_{\alpha \alpha} + l_{\alpha c}}{l_{\sigma 0}}$$  \hspace{1cm} (8a)$$

$$\frac{d i_b(u_1)}{d \tau} = -\frac{u_d}{l_{\sigma 0}} \frac{1}{l_{\alpha \alpha}}$$  \hspace{1cm} (8b)$$

$$\frac{d i_c(u_1)}{d \tau} = -\frac{u_d}{l_{\sigma 0}} \frac{1}{l_{\alpha c}}$$  \hspace{1cm} (8c)$$

The phase components $p_a, p_b$ and $p_c$ of the rotor bar position vector are introduced next, using $\alpha \beta \gamma$ definition (1) and referring to (4) and (5). The derivative $d i_a(u_1)/d \tau$ from (8a) is expressed as

$$\frac{d i_a(u_1)}{d \tau} = \frac{u_d}{l_{\sigma 0}} \frac{1}{1 + l_{\alpha \alpha} \cos(n p \theta) + l_{\alpha \alpha} \cos(n p \theta + \frac{2 \pi}{3})}$$  \hspace{1cm} (9)$$

Each of the two terms in the right can be written as a binomial expansion. The first term becomes

$$\equiv \frac{u_d}{l_{\sigma 0}} \left(1 - l_{\alpha \alpha} \cos(n p \theta) + \frac{l_{\alpha \alpha}^2}{2} \cos^2(n p \theta) + \ldots \right)$$  \hspace{1cm} (10)$$

<table>
<thead>
<tr>
<th>$p_a$</th>
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<tr>
<td>$-g \frac{d i_a(u_1)}{d \tau} + 2$</td>
<td>$-g \frac{d i_b(u_1)}{d \tau} - 1$</td>
<td>$-g \frac{d i_c(u_1)}{d \tau} - 1$</td>
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<td>$g \frac{d i_c(u_2)}{d \tau} + 2$</td>
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<tr>
<td>$-g \frac{d i_a(u_3)}{d \tau} - 1$</td>
<td>$-g \frac{d i_b(u_3)}{d \tau} - 1$</td>
<td>$-g \frac{d i_c(u_3)}{d \tau} + 2$</td>
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<tr>
<td>$g \frac{d i_a(u_4)}{d \tau} + 2$</td>
<td>$g \frac{d i_b(u_4)}{d \tau} - 1$</td>
<td>$g \frac{d i_c(u_4)}{d \tau} + 2$</td>
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<tr>
<td>$-g \frac{d i_a(u_5)}{d \tau} - 1$</td>
<td>$-g \frac{d i_b(u_5)}{d \tau} + 1$</td>
<td>$-g \frac{d i_c(u_5)}{d \tau} + 2$</td>
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<td>$g \frac{d i_b(u_6)}{d \tau} - 1$</td>
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<td>$-h \frac{d i_c(u_6)}{d \tau} + 1$</td>
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Table 1. Wye-connection

Table 2. Delta-connection
The second term has the same structure. Of the binomial expansions, all higher order terms can be neglected. This is justified since \((l_{\Delta}/\sigma_0)^2 = 0.062 = 0.0036\) for the induction motor used in the experiments. The remaining first-order terms are added, which then yields

\[
\frac{di_a(u_1)}{d\tau} = \frac{u_d}{\sigma_0} \left[ 2 - \frac{l_a}{\sigma_0} \cos(n_{tp}\vartheta) + \cos\left(\frac{n_{tp}\vartheta + 2\pi}{3}\right) \right]
\]

(11)

The derivatives \(di_a(u_1)/d\tau\) from (8b), and \(di_b(u_1)/d\tau\) from (8c), are treated in the same manner. Referring to (1), we have from (8b)

\[
\frac{di_b(u_1)}{d\tau} = -\frac{u_d}{\sigma_0} \cdot \frac{1}{1 + \frac{l_a}{\sigma_0} \cos(n_{tp}\vartheta)}
\]

(12)

\[
\approx -\frac{u_d}{\sigma_0} \left( 1 - \frac{l_a}{\sigma_0} \cos(n_{tp}\vartheta) + \left(\frac{l_a}{\sigma_0}\right)^2 \cos^2(n_{tp}\vartheta) \pm \ldots \right)
\]

The higher order terms of the Taylor expansion are again neglected. Equation (8c) is treated in the same fashion as (8b). From these calculations, the line current derivatives for excitation by \(u_1\) are obtained as

\[
\frac{di_i(u_1)}{d\tau} = \frac{u_d}{\sigma_0} \left( 2 + \frac{l_a}{\sigma_0} \cos(n_{tp}\vartheta - \frac{2\pi}{3}) \right)
\]

(13a)

\[
\frac{di_b(u_1)}{d\tau} = -\frac{u_d}{\sigma_0} \left( 1 - \frac{l_a}{\sigma_0} \cos(n_{tp}\vartheta) \right)
\]

(13b)

\[
\frac{di_i(u_1)}{d\tau} = -\frac{u_d}{\sigma_0} \left( 1 - \frac{l_a}{\sigma_0} \cos(n_{tp}\vartheta - \frac{4\pi}{3}) \right)
\]

(13c)

Making use of the definitions (4) and (5) permits deriving the phase components \(p_a, p_b\) and \(p_c\) of the rotor bar position vector

\[
p_b = \frac{l_{\sigma_0}}{u_d} \frac{di_b(u_1)}{d\tau} - 2
\]

(14a)

\[
p_a = \frac{l_{\sigma_0}}{u_d} \frac{di_a(u_1)}{d\tau} + 1
\]

(14b)

\[
p_c = \frac{l_{\sigma_0}}{u_d} \frac{di_i(u_1)}{d\tau} + 1
\]

(14c)

The phase components \(p_a, p_b\) and \(p_c\) at excitation by one of the other active switching states, \(u_2\) through \(u_6\), are derived in the same manner. The results are shown in Table 2, where the common factor

\[
h = \frac{l_{\sigma_0}}{u_d}
\]

(15)

is introduced.

Fig. 3. Switching state vectors and extended modulation sectors I through IV; only the portions marked by bold arcs are used as sectors II and IV in the overlapping regions.

### III. Acquisition of Position Signals

The equations listed in Table 1 and Table 2 have a common property: The phase components \(p_a, p_b\) and \(p_c\) of the rotor bar position vector not only depend on the respective current derivatives, but they exhibit an offset in addition. The offset values appear as integers, 1 or 2, respectively. Note that the position vector (4) does not have a dimension.

There are many ways to eliminate the offset from two or more acquired current derivative values. Good results were obtained by modifying the switching sequence of the pulse-width modulator whenever a rotor position signal needs to be acquired. This is done every 1 or 2 ms. In such case, the following conditions are set:

- Instead of executing the next half-cycle of the modulation within a 60°-sector, an extended 120°-sector is chosen.
- The on-state durations of the two active switching state vectors of the extended modulation cycle must have minimum values. This allows for the parasitic switching transients at the commutating inverter terminal to fade away before a sample of the current derivative is taken.

Given the six active switching state vectors shown in Fig. 3, a total of four 120°-sectors can be defined. Out of these, the particular 120°-sector is selected in which the reference voltage vector \(u^*\) resides. The vector \(u^*\) is the input variable of the controlling pulsewidth modulator.

To give an example, it is assumed that the reference vector \(u^*\) is in the extended modulation sector I, and a position signal must be acquired. Instead of the original half-cycle \(u_0(t_0/2) \ldots u_1(t_1) \ldots u_2(t_2) \ldots u_7(t_7/2)\), the extended modulation should be \(u_0(t_0/2) \ldots u_1(t_1) \ldots u_3(t_3) \ldots u_7(t_7/2)\). The notation associates to each switching state vector its on-duration in brackets.

If the on-state durations of \(u_1\) and \(u_3\) are less than the required minimum value \(t_{min}\), their on-times are extended to equal that minimum value. An extension of the on-times in-
terferes with the commanded complex volt-seconds of \( u^* \tau t_0 / 2 \), where \( t_0 / 2 \) is the half-cycle duration. A correction is then made by introducing the compensating switching states \(-u_1 = u_4\) and \(-u_2 = u_6\) for the respective durations \( u_1 t_{\text{min}} - t_1 \) and \( u_6 t_{\text{min}} - t_1 \). The extended modulation cycle is

\[
\{u_0\} \rightarrow \{u_1\} \rightarrow \{u_6\} \rightarrow \{u_7\} \rightarrow \{u_4\} \rightarrow \{u_3\} \rightarrow \{u_0\} \tag{16}\]

The switching states vectors are arranged such that the transitions between them require only one commutation. Fig. 4 shows the resulting logic output signals of the pulsewidth modulator. Extended modulation cycles as in (16) are applied every second time in a reverse direction to reduce the distortions of the machine currents. Postion signal sampling is done in the respective intervals of duration \( t_{\text{min}} \). The extended switching state vectors being \( u_1 \) and \( u_3 \) in this example.

Instead of computing the phase components \( p_a, p_b \) and \( p_c \) from these signals, the orthogonal components \( p_a \) and \( p_b \) of the rotor bar position vector can be directly determined. This can be done in various ways. Given a stator winding in wye connection, one of the possibilities is adding two selecting terms from Table 1 by taking

- for \( p_a \) the sum of the first term in row 1 and two times the third term in row 3

\[
p_a = p_a = \frac{1}{3} \left[ -g \frac{d_i(u_1)}{d\tau} + 2 + \left( -g \frac{d_i(u_3)}{d\tau} - 1 \right) \right] = \frac{g}{3} \left[ -\frac{d_i(u_1)}{d\tau} - 2 \frac{d_i(u_3)}{d\tau} \right] \tag{17a}\]

- and for \( p_b \) the difference between the third term and the second term, both in row 1

\[
p_b = \frac{1}{\sqrt{3}} \left( p_b - p_c \right) = \frac{1}{\sqrt{3}} \left[ -\frac{d_i(u_1)}{d\tau} + 1 + \frac{d_i(u_1)}{d\tau} + 1 \right] = \frac{g}{\sqrt{3}} \left[ -\frac{d_i(u_1)}{d\tau} + \frac{d_i(u_1)}{d\tau} \right] \tag{17b}\]

Table 3. Wye-connection

Switching sequences and components of the rotor bar position vector as functions of the stator current derivatives for extended modulation half-cycles in sectors I through IV

It is seen that the procedure eliminates the offsets from the measured current derivatives; the amplitude of the position vector \( p \) is defined by \( g \).

To obtain more accurate information from an extended switching sequence, an alternative approach is followed to exploit existing redundancies. Instead of using only two current derivative signals as in (17), the required information is averaged from the contributions of six derivatives. Table 3 shows how the components \( p_a, p_b \) of the rotor bar position vector \( p, (5) \) are computed from the current derivative samples of the extended modulation sequences in different sectors.

To derive the corresponding equations for a delta-connected stator winding, a comparison is made between (6) and (14) while referring to (7) and (15). The result is arranged as follows:

\[
p_a = -g \frac{d_i(u_1)}{d\tau} + 2 \quad \rightarrow \quad p_b = h \frac{d_i(u_1)}{d\tau} - 2 \tag{18a}\]

\[
p_b = -g \frac{d_i(u_1)}{d\tau} - 1 \quad \rightarrow \quad p_c = h \frac{d_i(u_1)}{d\tau} + 1 \tag{18b}\]

\[
p_c = -g \frac{d_i(u_1)}{d\tau} - 1 \quad \rightarrow \quad p_a = h \frac{d_i(u_1)}{d\tau} + 1 \tag{18c}\]

The subscript \( Y \) refers to wye-connected windings and the subscript \( \Delta \) to delta-connected windings. The following observations are made:

\[ 0 \]
1. Similar equations show up when arranging the phase values of the rotor bar position vector for delta-connection and wye-connection in a cyclically changed order: $a \rightarrow b$, $b \rightarrow c$, $c \rightarrow a$.
2. The signs of the respective expressions are then reversed.
3. The current derivatives are weighed by different gain factors, $g$ and $h$, respectively. These will result in different amplitudes when computing the respective rotor bar position vectors. Table 3 shows that the amplitudes are proportional to only the sampled derivative values since all integers in (18) are eliminated. The amplitude of the rotor bar position vector carries no information.

Accounting for these differences, the equations in Table 3 can be also used for position estimation of delta connected machines. Even more, conditions (1) and (2) above indicate only phase angle displacements. These are of no importance as only the increments of the position signals are processed. As a consequence, the algorithm in Table 3 applies for both wye and delta connected machine windings, as the amplitude differences of condition (3) are insignificant. It is obvious, then, that any machine can be used for sensorless position estimation, not knowing whether the windings are in wye or delta connection.

IV. EXPERIMENTAL RESULTS

A. Measurement of the current derivatives

Two established methods exist for measuring the derivative $di/dt$ of the machine current. Taking the difference $\Delta i$ of two current samples within a time interval $\Delta t$ leads to $di/dt \approx \Delta i/\Delta t$. The method requires much higher resolution of the analog-to-digital conversion since small differences of high amplitude values must be transmitted. It also gives erroneous results when the current samples are corrupted by noise. An alternative is using a Rogowski coil, which is wound toroidally around the current conductor. The coil windings link with the magnetic field produced by the current. The induced voltage is $u_r = d\psi/dt = l_m di/dt$, where $\psi$ is the flux linkage and $l_m$ is the mutual inductance.

To obtain an output voltage of required magnitude, the number of toroidal turns must be sufficiently high. The voltage increases in proportion, however the winding inductance with the square of the number of turns. This strongly increases the tendency that parasitic oscillations are created at changes of $di/dt$, an effect that is further promoted since the winding capacitances also increase. Different Rogowski coil arrangements were tested. Their natural frequencies and damping vary with the coil geometry and the number of turns. Examples are shown in Fig. 5(a) and Fig. 5(b). Damping resistors reduce the oscillations, but make the response sluggish; also the output voltage gets too much reduced.

A better solution was found using the air-cored coaxial transformer shown in Fig. 6. Only a few turns of coaxial cable are required. The respective phase current is conducted by the outer shield; the inner conductor provides a $di/dt$-signal of sufficient amplitude. The low capacitance of such winding arrangement avoids that parasitic oscillations are excited.
The response is shown in Fig. 5(c).

B. Amplitude error compensation

Fig. 7 shows an oscillogram of the phase current $i_a$ and its measured derivative signal $di_a/dt$. Although the current signal in the upper trace looks fairly smooth, its derivative signal shows two oscillatory components superimposed on it: (i) a high-frequency component that originates from wave reflections on the motor cable, and (ii) a low-frequency component caused by wave reflections within the stator winding of the machine [5]. The large amplitude of the high-frequency signal is seen to be partially blanked out by a limiter circuit.

Sampling of the current derivative signal takes place when the high-frequency oscillations have decayed, and when about the maximum amplitude of the low-frequency oscillation occurs thereafter, which is at $t_s = t_0 + t_{min}$, where $t_0$ is the beginning of the inverter commutation, Fig. 7.

The amplitude value of the low-frequency oscillation at the sampling instant $t_s$ is a disturbance. It introduces an error to the sampled derivative signal, which in turn produces dc-component errors of the sampled signal. Fig. 8(a) shows that the offset values of the four recorded derivative signals do not relate as $g$ and $2g$ as they should according to Table 1, or as $h$ and $2h$ according to Table 2. Their deviations are compensated by dividing each signal by its own absolute mean value, and multiplying the result by either $g$ and $2g$ (or by either $h$ and $2h$). As shown in Fig. 8(b), the multiplications restore the correct relations between the offset values. However, these values cancel anyway when computing the components of the position vector, as is evident from (17). Therefore, the multiplications introduce just a scaling factor which can be arbitrarily chosen, if needed, to ensure accurate numerical representation.

C. Position signals

Measured signals that represent the components of the stator current derivative vector $di_s/dt$ are shown in Fig. 8(b). They were recorded at zero stator frequency. The current derivative values where sampled at 1 kHz, applying the extended switching sequence (16) in sector I. The signals of the upper two traces in Fig. 8(b) define $p_a$ and $p_c$ according to the equations in the first row of Table 1, while the waveforms in the lower two traces make up for $p_b$ and $p_a$ as indicated by the expressions in the third row of Table 1.

The current derivative values serve to compute the components $p_a$ and $p_b$ of the rotor bar position vector $p_r$ according to the equations in Table 3. The resulting waveforms are shown in Fig. 9. In our setup, a superimposed component of double slip-frequency was encountered, particularly visible in the trace $p_\alpha$. The effect is interpreted as an inherent aniso-
The anisotropy of the rotor. The vector filter shown in Fig. 11 eliminates the disturbing effect.

Fig. 10 shows the same signals as in Fig. 9, but without amplitude error compensation. It is obvious that the corresponding trajectory of the rotor bar position vector (5) is an ellipse displaced from the origin. The estimated rotor position would be much in error.

The position signals obtained with amplitude error compensation are shown in the upper two traces of Fig. 12. They define the rotor position angle as displayed in the lower traces. In a practical implementation, the rotor position \( \vartheta \) within a full revolution is obtained by incrementing (or decrementing at reversed rotation) a modulo-\( n_{rp} \) counter whenever a full revolution of \( p_r \) is completed. Hence, the incremental rotor position within a full revolution is

\[
\vartheta = (2\pi C_0 + \vartheta_t)/p_{n_{rp}} \tag{19}
\]

where \( C_0 \) is the state of the counter.

Fig. 13 demonstrates that the position error is less than 0.5° mechanical. The two spikes in the lower curve appear only in this comparison. They are caused by the 360°-periodicity of the numerical representation. Finally, Fig. 14 gives proof of the insensitivity to parameter variations: The dc link voltage is reduced to one half of its nominal value without affecting the estimation accuracy of the rotor position angle. The same insensitivity exists against variations of the average inductance \( l_{\sigma 0} \) which is confirmed by (7), or (15).

C. Motor data

The induction motor used for the experiments has the following data: \( P_R = 4.5 \text{ kW}, U_R = 360 \text{ V}, 50 \text{ Hz}, I_R = 9.26 \text{ A}, \cos \varphi = 0.86, 1730 \text{ rpm}, 2p = 4, n_{rp} = 22, R_s = 1.026 \Omega, R_r = 0.639 \Omega, L_s = 4.48 \text{ mH}, L_r = 6.29 \text{ mH}, L_m = 140 \text{ mH}, \) no skew.

V. SUMMARY

The anisotropy of the cage rotor of an induction motor influences the stator current derivatives during the transient conditions that follow the turn-on of an active switching state vector. The phenomenon enables a quasi-instantaneous estimation of the absolute angular position of the rotor within a rotor bar pitch. Also the incremental angular position of the rotor is then defined since the number of rotor bars per pole pair is known. A single algorithm serves to determine the rotor bar position angle of an induction motor, regardless of whether the stator windings are wye or delta connected. Since the current derivatives are acquired at the inverter terminals, no additional wires are required between the inverter and the motor.

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VI. REFERENCES


**Figure Captions**

**Fig. 1** Inverter topology in switching state $u_1$ feeding an induction motor with (a) wye-connected, (b) delta-connected stator windings; note that there is no additional connection to the motor.

**Fig. 2** Stator winding connections

**Fig. 3** Switching state vectors and extended modulation sectors I through IV; only the portions marked by bold arcs are used as sectors II and IV in the overlapping regions.

**Fig. 4** Schematic switching sequence of an extended modulation cycle as per (16); $d_a, b, c$ are the logic output signals of the pulselength modulator

**Fig. 5** Response of different transducers for $d\text{ildt}$-measurement; (a) and (b): different Rogowski coils, (c) the coaxial transformer shown in Fig. 5.

**Fig. 6** The coaxial transformer used for $d\text{ildt}$-measurement

**Fig. 7** Measured stator current and stator current derivatives at turn on of the switching state vectors $u_1$. The damped low-frequency oscillation in the lower curve is caused by wave reflections in the machine winding

**Fig. 8** Measured signals of the stator current derivatives at turn on of the switching state vectors $u_1$ (upper two traces) and $u_3$ (lower two traces); (a) without, (b) with amplitude error correction; zero stator frequency, slip frequency 1 Hz

**Fig. 9** Orthogonal components of the rotor position vector $p'$ derived from the signals in Fig. 6; the offset is eliminated

**Fig. 10** Orthogonal components of the rotor position vector $p'$ derived from the signals in Fig. 6; the offset is eliminated

**Fig. 11** Filter to eliminate the influence of a mechanical eccentricity of the rotor

**Fig. 12** From top: position vector components $p_a$ and $p_b$, estimated position angles $n_{10} \hat{\theta}$ and $\hat{\theta}$, recorded at zero stator frequency, slip frequency 0.6 Hz

**Fig. 13** From top: measured and estimated position angles $\theta$ and $\hat{\theta}$, position angle error $\Delta \theta$, recorded at zero stator frequency, slip frequency 1 Hz

**Fig. 14** Insensitivity against parameter variations. From top: position vector components $p_a$ and $p_b$, estimated position angle $\theta$, dc link voltage $U_d$, recorded at zero stator frequency, slip frequency 1 Hz

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**Table 1. Wye-connection**

| Phase components of the rotor bar position vector as functions of the stator current derivatives following the turn-on of the active switching state vectors $u_1$ through $u_6$ |

**Table 2. Delta-connection**

| Phase components of the rotor bar position vector as functions of the stator current derivatives following the turn-on of the active switching state vectors $u_1$ through $u_6$ |

**Table 3. Wye-connection**

Switching sequences and components of the rotor bar position vector as functions of the stator current derivatives for extended modulation half-cycles in sectors I through IV