High-Performance Current Regulation and Efficient PWM Implementation for Low Inductance Servo Motors

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Abstract — The paper reports on a standard microcontroller implementation of a pulsewidth modulator and near-deadbeat current regulator for high switching frequency. The application is in high-performance positioning systems. The control strategy relies on a simplified machine model without incurring performance degradations. Changes between different modulation strategies are programed depending on the modulation index. The values of switching time durations are obtained exclusively by decision making, thus minimizing computational load. Features like overmodulation, dynamic overmodulation, anti-windup, and reduction of switching frequency at thermal overload are included.

1. INTRODUCTION

Low inertia permanent magnet (PM) synchronous motors are used with preference in high-performance positioning systems. Owing to a large magnetic airgap, the leakage inductance is very low which favors a fast current rise. It makes the design of the current control system critical on the other hand. The problem is even more pronounced with the upcoming generation of ironless linear servo motors. Such motors will be increasingly used in future machine tool applications.

The performance degradations of existing current regulators when applied to low inductance machines are due to deficiencies of the pulsewidth modulation (PWM) methods, and shortcomings in the structural design of the current control system. Regarding PWM, the low leakage inductance requires operation at high switching frequency, typically $f_s = 16 - 48$ kHz. As digital solutions require DSP computing power, analog modulation techniques are mostly preferred [1]. The drawbacks of analog current regulators of the triangular intersection category [2] are: the phase lag error at higher speed; reduced gain at steady-state overmodulation [3]; poor $d$-$q$ decoupling; poor dynamic performance at dynamic overmodulation; and complex hardware for anti-windup [2]. Analog controllers cannot handle different inductance values in the $d$- and $q$-axis. Hysteresis type current controllers satisfy these requirements. They exhibit poor harmonic characteristics instead which makes them inadequate for high-performance applications. Complex hardware is required to maintain the switching frequency constant [4].

The alternative is digital current regulation. A preferred approach uses PI-controllers in synchronous coordinates. The method exhibits only moderate transient performance, lack of dynamic decoupling, and parameter dependency [3]. Alternative methods like deadbeat current control [5] and predictive current control [6] require accurate models of the machine, and of the overmodulation nonlinearities. This adds complexity to the digital algorithms and requires high computing power. Although the use of floating point DSP facilitates the estimation of parameter variations [7], hardware cost contributes considerably to the total product price. As high-performance positioning is mostly a low-power application, only standard microcontroller solutions will be competitive in this particular market segment.

This paper presents a solution that avoids the critical trade-off between high-performance requirements and hardware complexity, particularly for applications requiring low leakage inductance PM synchronous motors, or linear positioning drives. The first section describes a high-switching frequency pulsewidth modulator, also suitable for FPGA implementation. The modulator features full overmodulation capability, and a reduction of switching frequency to prevent thermal overload of the power converter, both without compromising on dynamic performance. The second section describes a near-deadbeat current regulator. Performance issues such as dynamic decoupling, transient overshoot, and anti-windup are addressed.

2. PULSEWIDTH MODULATION BY DECISION MAKING

2.1 Principle of operation

For the implementation of a digital space vector modulator under time critical conditions, on-line computation of the switching durations has been earlier replaced by retrieving stored information [8]. The approach described in this paper goes one step further. Instead of storing precalculated on-state durations in ROM-tables, the components of the voltage reference vector are classified in a decision tree. Their respective signs and magnitudes determine both the on-state durations, and the time sequence of the switching state vectors at varying modulation methods. The switching frequency $f_s$ is held constant.

Fig. 1 shows that the decision procedure is started by entering the transformed reference vector $u^*_{\text{flat}} = u_{\alpha} + ju_{\beta}$ into a first structure, called decision tree I. A preliminary set of the on-state durations $t_a$ and $t_b$ is determined here, and the sector that is addressed by $u^*_{\text{flat}}$ is identified. The decision route is then fanned out, depending on which of the six sectors is addressed. Each sector has a different decision tree in this sub-
sequent stage II. These decision trees provide the information to generate the switching sequence, namely

- the final on-state durations \( t_a, t_b, t_0 \)
- the 30°-segment in which the actual voltage reference vector \( u^* \) is located, and
- the modulation mode. Two different modulation modes are used in the linear range. At higher voltage, two overmodulation modes [10], a bang-bang mode, and the six-step mode can be selected, depending on the actual operating condition.

The on-state durations are subsequently loaded to a timer circuit. A state machine generates the inverter control signals \( u_k \). It also delivers a pulse train of frequency \( 2f_s \). The pulses are exactly placed in the center of the zero vector time interval to enable sampling the fundamental component of the distorted load current.

2.2 Modulation algorithm

The modulation algorithm is derived with reference to the first 60°-sector in the complex plane, shown in Fig. 2.

This sector is addressed whenever \( 0 \leq \arg(u^*) \leq \pi/3 \), where \( u^* \) is the reference voltage vector. During each modulation subcycle of duration \( T_0 \), a switching sequence is generated, composed of three switching state vectors \( u_1(t_1), u_2(t_2) \), and \( u_0(t_0) \), where \( t_1, t_2, t_0 \) are the on-state durations of the respective switching state vectors \( u_1, u_2, u_3 \).

Considering the two active switching state vectors \( u_1 \) and \( u_2 \) in Fig. 2 as the coordinate axes, the reference voltage vector \( u^* \) can be decomposed as

\[
    u^* = u_1^* + u_2^*,
\]

where the vectors \( u_1^* \) and \( u_2^* \) coincide with the respective components in the axis directions.

The active switching state vectors \( u_1 \ldots u_6 \) are considered normalized to have unity magnitude. The normalizing voltage is then \( 2/3 U_d \), where \( U_d \) is the dc link voltage. Still referring to the first sector, the stator voltage space vector, averaged over one subcycle \( T_0 \), is

\[
    u_s = (t_1u_1 + t_2u_2)/T_0.
\]

The modulation law requires the actual stator voltage vector \( u_s \) to equal its reference value \( u^* \)

\[
    u_s = u^* = \Re\{u^*\} + j\Im\{u^*\},
\]

where \( \Re\{u^*\} \) and \( \Im\{u^*\} \) are the Cartesian coordinates of \( u^* \) in the stationary reference frame. Using \( u_1 = 1 \) and \( u_2 = \exp(j\pi/3) \), equations (2) and (3) are solved for the switching durations

\[
    t_1 = T_0 \left( \Re\{u^*\} - \frac{1}{\sqrt{3}} \Im\{u^*\} \right) \tag{4a}
\]
\[
    t_2 = T_0 \frac{2}{\sqrt{3}} \Im\{u^*\} \tag{4b}
\]

As long as the linear modulation range is addressed, the condition \( t_1 + t_2 \leq T_0 \) holds. The subcycle is then completed by inserting a zero vector \( u_0(t_0) \) for the remaining time

\[
    t_0 = T_0 - (t_1 + t_2) \tag{4c}
\]

The zero vector does not contribute to the volt-second balance in (2).

The foregoing equations apply for the special case where the reference voltage vector is located in the first 60°-sector. Generally, the vector may be encountered in any of the six sectors. The general problem is solved by rotating \( u^* \) anticlockwise by an angle \( (S - 1)\pi/3 \), where \( S \in 1 \ldots 6 \) is the sector number. This rotation locates any reference vector in the first sector.

The rotated reference vector is inserted in (4) to compute the switching durations. A reverse rotation is then performed to determine the switching state vectors for inverter control. These are \( u_a = u_1 \exp(j(S - 1)\pi/3) \), and \( u_b = u_1 \exp(j\pi/3) \). In effect, \( u_a \) and \( u_b \) are the two switching state vectors located adjacent to \( u^* \).

For further simplification, the respective on-state durations of the general switching state vectors \( u_a \) and \( u_b \) are expressed as a percentage of the subcycle duration \( T_0 \).

\[
    t_a = t_1/T_0, \tag{5a}
\]
\[
    t_b = t_2/T_0. \tag{5b}
\]
and the reference voltage vector $u^*$ in (3) is subjected to the transformation

$$u_{\text{flat}}^* = u_\alpha + ju_\beta = \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{\sqrt{3}} \end{bmatrix} u^*. \tag{6}$$

The result of this transformation is denoted as $u_{\text{flat}}^*$, so as to indicate that the regular hexagon, defined by the switching state vectors in Fig. 3(a), gets flattened as shown in Fig. 3(b). The transformation offers the following advantages:

- The transformed sectors are separated by 45°-lines. This simplifies the sector identification as only the magnitudes of $u_\alpha$ and $u_\beta$ need to be compared.
- The numerical evaluation of the on-state durations gets reduced to a single addition. Referring again to the first sector, $S = 1$, the on-state durations of the active switching state vectors are obtained from (3) - (6) as
  $$t_a = u_\alpha - u_\beta \tag{7a}$$
  $$t_b = u_\beta + u_\alpha \tag{7b}$$

Similar solutions exist for the other sectors. They are displayed in the right-hand portion of Fig. 4(a).

The decision tree shown in Fig. 4 represents the modulation algorithm as derived from the foregoing equations. Its condensed version Fig. 4(b) illustrates that $u_{\text{flat}}^* = u_\alpha + ju_\beta$ is the input variable, depending on which the on-state durations $t_a$ and $t_b$, and the sector information $S$ are obtained.

Let us assume, for example, that the transformed reference voltage vector is located in sector 6: $u_{\text{flat}}^* = 0.5 - j 0.3$. The first condition in Fig. 4(a) is then true, thus identifying the right half plane in Fig. 3. The second condition is false, thereby selecting $u_\alpha \geq u_\beta$ as the third condition, which is true. Hence we have identified sector 6, and the switching durations are $t_a = -u_\alpha - u_\beta$ and $t_b = u_\alpha + u_\beta$. These two simple terms represent all of the arithmetics of the modulation process.

Note that the transformation (6) into the flattened reference frame Fig. 3(b) is only a particular way of doing the signal processing. It does not change the physical effect on the machine of the inverter switching states. The benefit is that the sector information and the switching times are directly obtained from the voltage reference vector. The implemented algorithm is still that of the original space vector modulation [9].

2.3 Switching sequences

The sequence of switching states within a subcycle determines the harmonic content of the machine currents, and also the switching frequency. The space vector modulation method (SVM) gives the best results at lower modulation index. It is characterized by the switching sequence

$$u_0(t_0/2) \ldots u_1(t_1) \ldots u_2(t_2) \ldots u_7(t_7/2) \tag{8a}$$

in any first, or generally in all odd subcycles, and

$$u_7(t_7/2) \ldots u_2(t_2) \ldots u_1(t_1) \ldots u_0(t_0/2) \tag{8b}$$

for the next, and all even subcycles. The notation in (8) associates to each switching state vector its on-duration in brackets. Each subcycle is composed of three switching transitions. The zero vector, represented in (8) by $u_0$ or $u_7$, is shared between two subsequent subcycles to permit sampling of the undistorted fundamental current space vector at the modulator clock rate $2/f_s$, [10].

The normalized harmonic distortion $d$ at SVM is plotted as a function of the modulation index in Fig. 5.
An alternative switching sequence the modified space vector modulation (M-SVM). It requires only two switching transitions per subcycle. Considering again the first sector, the switching sequences within the lower segment, defined by \(0 < \arg(u^*) < \pi/6\), are

\[
\mathbf{u}_0(t_0/2) \ldots \mathbf{u}_1(t_1) \ldots \mathbf{u}_2(t_2/2), \quad (9a)
\]
\[
\mathbf{u}_3(t_3/2) \ldots \mathbf{u}_4(t_4) \ldots \mathbf{u}_5(t_5/2), \quad (9b)
\]

for odd and even subcycles, respectively. The switching sequences are different in any upper segment: \(\pi/6 \leq \arg(u^*) \leq \pi/3\) in the first sector, and generally \(\pi/6 \leq \arg(u^* - (S - 1)\pi/3) \leq \pi/3\). In any upper segment, the active switching state vectors \((\mathbf{u}_1 \text{ and } \mathbf{u}_2)\) in (9) exchange positions within the sequence.

A switching sequence in M-SVM has less switching transitions. It enables a reduction of the switching frequency to 66\% as compared with SVM, when the subcycle duration \(T_0\) is not changed. It is an advantage of M-SVM that the harmonic distortion decreases at higher modulation index. The effect is displayed in Fig. 5.

This favorable property is exploited whenever the power devices of the inverter are driven towards their margin of thermal overload. The switching frequency, and hence the inverter switching losses, are then reduced by changing from SVM to M-SVM. The modulation index \(m_c\) at which the crossover between SVM and M-SVM takes place, is reduced when the inverter temperature increases beyond a given limit. The temperature itself is not measured. Instead, a thermal 1st-order model is implemented in software to represent the thermal time constant of the power devices. Their temperature is estimated based on the machine currents and the type of modulation.

The signal flow diagram Fig. 6(a) shows how the preferred switching sequence (SVM or M-SVM) is selected. Referring for example to the lower segment of any sector, and assuming that the modulator operates in the linear range \(t_a + t_b = t_{ab} < 1\), the condition \(t_{ab} > m_c\) decides between SVM and M-SVM. The crossover value \(m_c\) of the modulation index \(m\) is obtained from the thermal model. The value \(m_c\) is high at low inverter temperature, and vice versa.

### 2.4 Overmodulation

Overmodulation occurs when the reference voltage vector points to a location outside the hexagon limits in Fig. 3(a). The voltage capability of the inverter is then fully exploited, and a switching sequence that equals the reference voltage vector within a subcycle average cannot be delivered. Overmodulation trades a phase angle error against a higher voltage magnitude. A higher voltage is always available unless the reference voltage vector is exactly centered between the hexagon corners. The trade-off leads to increased lower order harmonics of the machine currents.

The upper limit of the overmodulation range is the six-step mode, where the switching frequency equals the fundamental frequency; both the modulation index \(m\) and the distortion factor \(d\) are then unity by definition [10]. A derivation is given in the Appendix.

The decision tree in Fig. 6(a) shows that overmodulation (OVM) is activated by the condition \(t_a + t_b = t_{ab} > 1\). This requires revising the original on-state durations, as the total on-time cannot exceed the subcycle duration. Note that the normalized subcycle duration is unity.

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**Fig. 5:** Distortion factor \(d\) versus modulation index \(m\), shown for space vector modulation and for modified space vector modulation.

**Fig. 6:** Decision tree II; the structure adds the functionalities of modified space vector modulation, overmodulation, and bang-bang control to the space vector modulator in Fig. 4; (a) signal flow diagram, (b) single-block representation.
The decision strategy is illustrated with reference to Fig. 7. Outside the linear modulation range, confined by the outer hexagon in Fig. 7, overmodulation gets selected whenever the voltage reference vector enters one of the hatched triangular areas. The other outside areas, which are shaded in Fig. 7, refer to bang-bang control, or to the six-step mode.

Also at overmodulation is the principle of a decision-based modulation algorithm observed. The original switching durations \( t_a \) and \( t_b \), as taken from decision tree I, are revised to fit into the subcycle interval. The condition is \( t_b + t_a = 1 \). Assuming for example that the voltage reference vector is located in the lower segment of sector 1, the revised value is determined by moving \( u^* \) along the dashed line in Fig. 7(a) towards the hexagon edge. In terms of the original \((u_1,u_2)\)-coordinate system (1), the original on-state duration \( t_b \) changes to \( t_b = 1 - t_a \), while \( t_a \) is maintained at its original value to complete the remaining portion of the subcycle duration.

Fig. 7(b) illustrates the corresponding situation in the upper segment of sector 1. It is obvious that the component \( u_{2^*} \) in (1) is now maintained at its original value, while the duration \( t_1 \) (normalized value \( t_a \)) fills the remaining time of the subcycle. Hence \( t_b = 1 - t_a \) is the entry in the flow diagram Fig. 6(a) for overmodulation in an upper segment.

While the magnitude of the effective stator voltage vector \( u_s \) is increased, a phase-angle error is introduced. Fig. 8 shows the relationship between the two phase angles \( \arg(u_s) \) and \( \arg(u^*) \). The control characteristics are identical to what is obtained by much more complex modulation methods [2].

### 2.5 Bang-bang control

The bang-bang sectors in Fig. 7 are selected when \( t_a > 1 \) in the lower segment, or when \( t_b > 1 \) in the upper segment. The respective entries in the decision tree Fig. 6 can be directly verified from an inspection of Fig. 7.

When a bang-bang sector is addressed, the selected switching state vector is turned on for the full subcycle. Should this occur in a dynamic condition at low fundamental frequency,
the same switching state vector, for example \( u_1 \), might continue to get selected during subsequent subcycles as illustrated in Fig. 9. Since the phase angle of \( u^* \) is generally different from that of \( u_1 \), a volt-second unbalance caused by the phase angle error is likely to build up. The error is integrated by the superimposed PI controller, and \( u_2 \) will get eventually selected for a number of subsequent subcycles. The resulting effect is a drastic reduction of the switching frequency. The stator voltage phase angle is still under full control, while the voltage magnitude is anyway at its maximum value.

The bang-bang mode offers two advantages:
- The inverter switching losses are reduced because the switching frequency reduces to less than one third of the original value. This entails less thermal stress in the inverter and a higher current amplitude is then permitted. High currents are particularly beneficial in a transient condition.
- The volt-second loss caused by inverter deadtime is reduced and hence the available fundamental voltage increases.

2.6 The six-step mode

The bang-bang condition of the modulator may also invoke the six-step mode. Six-step operation gets automatically established when the voltage demand increases at higher fundamental frequency. Other than the bang-bang mode, which is associated to a dynamic condition, the six-step mode gets enabled at near steady-state operation. The pulsewidth modulator itself exerts no influence on the selection between bang-bang operation and the six-step mode. It is the current control system, rather, which determines the type of modulation. To achieve this behavior, the current control system must be adequately designed.

3. The Current Control System

3.1 Machine model

The dynamic analysis of the PM machine is based on complex state variables [5, 11]. Space harmonics and rotor eddy currents are neglected. The voltage equation in rotor coordinates is

\[
\mathbf{u}_s = r_{\mathbf{i}_s} + l_{\mathbf{i}_s} \frac{d\mathbf{i}_s}{dt} + j\omega \mathbf{\psi}_m.
\]  

(10)

where \( \mathbf{u}_s \) is the stator voltage, \( \mathbf{i}_s \) is the stator current, \( r_s \) is the winding resistance, \( \omega \) is the angular velocity of the rotor, and \( \mathbf{\psi}_m \) is the stator flux linkage vector. The magnetic saliency of the machine is modeled in (10) by the inductance tensor

\[
\mathbf{l}_s = \begin{bmatrix} l_d(i_d) & 0 \\ 0 & l_q(i_q) \end{bmatrix}
\]  

(11)

The direct and quadrature inductances, \( l_d \) and \( l_q \) in (11), depend strongly on magnetic saturation. The stator flux linkage is

\[
\mathbf{\psi}_s = l_s * \mathbf{i}_s + \mathbf{\psi}_m.
\]  

(12)

where the magnet-induced flux linkage \( \mathbf{\psi}_m \) has only a \( d \)-axis component. The contribution of the stator current vector \( \mathbf{i}_s \) to the stator flux linkage depends on its phase displacement from the \( q \)-axis and on the degree of saliency. Both contributions are correctly modeled by the expression \( l_s * \mathbf{i}_s \) in (10) and (12).

The differential equation for the stator current vector

\[
\tau_s * \frac{d\mathbf{i}_s}{dt} + \mathbf{i}_s = -j\omega \tau_s * \mathbf{i}_s + \frac{1}{l_s}(\mathbf{u}_s - j\omega \mathbf{\psi}_m)
\]  

(13)

is derived from (10). Its graphic representation is the signal flow diagram Fig. 10. The term \( j\omega \mathbf{\psi}_m \) in (13) represents the rotor induced voltage. The shaded area on the right-hand side of Fig. 10 contains the dynamics of the mechanical system, being described by the differential equation

\[
\tau_m \frac{d\omega}{d\tau} = \mathbf{\psi}_s \times \mathbf{i}_s \bigg|_z - T_L,
\]  

(14)

where the electromagnetic torque \( T_e \) is obtained as the \( z \)-component of the vector product. The load torque is \( T_L \), and the mechanical time constant is \( \tau_m \).

The design of a current control system for the structure in Fig. 10 is targeted at an implementation in standard microcontroller hardware, however without incurring compromises as regards the dynamic performance. This objective was achieved by dispensing with the customary back-emf observ-
er and disturbance estimator [7]. Further, an extremely simplified machine model was used, in which the steady-state stator induced voltage $j\omega \Psi_s$ from (10) and (12), and the resistive stator voltage drop $r_{sd}s$, are treated as unmodeled disturbances

$$u_{\text{dist}} = j\omega (l_s s_i + \Psi_m) + r_s i_s \quad \text{(15)}$$

The subscript classifies this voltage as a disturbance. This disturbance gets compensated in good approximation by the output signal $u_1$ of an error integrator: $u_1 + u_{\text{dist}} = 0$. The error integrator forms part of the current control system in Fig. 12. If the integrator time constant is small, the disturbance does not exert a noticeable effect on the system behavior, and the differential equation (13) reduces to

$$\tau_s \frac{d i_s}{d\tau} = \frac{1}{r_s} (u_s + u_1 - u_{\text{dist}}) = 0 \quad \text{ (16)}$$

The associated signal flow graph in the shaded area of Fig. 11 represents the simplified machine model as an integrator, the time constant of which has the quality of a tensor:

$$\tau_s = \frac{l_s}{r_s}. \quad \text{ (17)}$$

The time constant tensor ensures that a step voltage input of arbitrary orientation in space generates a change of the output current vector in a different orientation. The effect is a consequence of the magnetic saliency. Saliency produces the tendency of the current vector to move closer to either the positive, or the negative $d$-axis, whichever is nearer.

3.2 The current controllers

The conceptual design of the current control system aims at achieving deadbeat behavior within one subcycle interval $T_0$ of the pulsewidth modulator. A solution to this problem was described in an earlier publication [12]. Time delay elements in the controller were used to compensate for a large time delay in the plant. The approach has been found expedient to deal with the time delay introduced by digital signal processing in the present application. The controller responds to dynamic changes by supplying a defined volt-seconds pulse to the machine, thus forcing the current vector to the desired location in a fast process. Against this, the unmodeled disturbances (15) have only low rates of change. They do not interfere with the control which justifies the use of the simplified machine model Fig. 11.

The current control system is implemented in rotor coordinates. Its feedback signal is the fundamental current $i_{s1}$, obtained from the distorted load current by zero-vector center time sampling. The arrangement is shown in the lower right portion of Fig. 12.

The voltage required to nullify a current error $\Delta i_s$ within a subcycle time $T_0 = 1/2f_s$ is obtained from (16) and (17):

$$\Delta u_s = \frac{1}{T_0} (l_s \Delta i_s) \quad \text{ (18)}$$

The corresponding volt-second value $\Delta u_s T_0$ is generated by the deadbeat controller on the left-hand side of Fig. 11. Following the sampling process, a current error $\Delta i_s$ produces the voltage $\Delta u_s$ for the duration of one subcycle. The signal is delayed by $T_0$ and subsequently subtracted from the output. It eliminates the original signal exactly when the current error has been eliminated by the deadbeat action.

The unmodeled disturbances may not get perfectly canceled, and hence only near-deadbeat performance can be expected.

In a continuous-signal representation, the deadbeat algorithm models as an ideal differentiator. When added to an existing $P$ controller, a PD controller is obtained; the differentiating time constant of the $D$-channel is $T_0$. Such controller forms part of the current control system, shown as the $PD$ channel in Fig. 12.

The required long-term accuracy is achieved by the error integrator in the $I$-channel. The input to the error integrator is delayed by one sampling interval $T_0$, since all commanded changes are executed, and external disturbances are compensated, within that very time $T_0$ by the near-deadbeat algorithm.

In the steady-state, the integrator output is constant and equals the stator voltage of the machine.

The error integrator combines as a $PI$ controller with the aforementioned $P$-channel. The two channels of the $PI$ controller are separated in the graph Fig. 12 as they receive different input signals. In accordance with the simplified machine model in Fig. 11, also the contribution of the $P$-channel to the controller output signal depends on the inductance tensor $l_s$, thus taking care of the magnetic saliency of the machine.
The design of the PI parameters takes into account that this controller is relieved from handling transient conditions by the parallel near-deadbeat channel. Also, the near-deadbeat channel is inactive in the steady-state. Hence the PI controller can be designed to establish the stability of a system that is not subjected to large-signal changes. This condition permits selecting a very small integrator time constant $T_i$ for better disturbance rejection. Particularly the error caused by unmodeled structures of the simplified machine model, having adverse effects on the deadbeat performance in principle, gets extremely fast compensated by the integrator signal. The experimental results presented in Section 4 demonstrate that the compensation is almost perfect. Hence the controller assumes deadbeat performance in effect.

### 3.3 Signal limiters

The control structure Fig. 12 contains three complex signal limiters L1, L2, and L3.

Limiter L3 is a standard element in the I-channel of any PI controller. It constrains the integrator output to the maximum stator voltage that can be supplied by the inverter at unity modulation index.

The purpose of this limiter is better understood when looking at a PI controller without signal limiter. Large-signal current changes would then drive the integrator output to high values, and hence the modulation index beyond unity. The modulator would saturate, and a persisting current error would let the error integrator output further increase (windup). A negative current error, i.e. an undesired current overshoot, would be produced to reset the error integrator. It is important, therefore, to stop the error integration whenever the modulator saturates.

Merely limiting the integrator output does not solve the windup problem since saturation of the modulator is nonlinear. The voltage limit is given by the hexagonal structure in Fig. 3. The use of a modulator model [2] would add to the complexity of the system. A simple solution is the $q$-axis limiter L1 at the input of the error integrator in Fig. 12. The limiter prevents an excessive increase of the integrator output in the presence of long-term errors.

While the current control system operates with an unsaturated modulator, limiter L1 has no impact on the performance since the near-deadbeat characteristic of the D-channel eliminates large-signal errors. Hence the PI controller receives only small-signal errors. It operates as a linear controller since the limiters do not interfere. Its purpose just reduces to maintaining small-signal stability, as previously explained.

Limiter L2 gets only activated in the overmodulation mode.

### 3.4 Current control at overmodulation

The upper limit of the overmodulation range is the six-step mode which produces highly distorted current waveforms. The trajectory of the stator current vector in this operating mode is shown in Fig. 13. It is plotted in rotor coordinates. Its shape demonstrates that the current harmonics are particularly large in the $d$-axis, which is an inherent characteristic of the six-step mode, and also of bang-bang operation. The high $d$-current harmonics should be therefore considered a desired effect. They must not be inhibited by a compensating action of the current controller.

This purpose is served by the $d$-signal limiter L2 in the dead-
beat (PD-) channel of the current controller. Bang-bang control is normally required during a transient condition in the base speed range. The transient voltage demand is then very high, and the linear modulation range is exceeded. The best reaction in this situation is maintaining the selected active switching state vector in the on-state, possibly for more than one subcycle.

The torque-building $q$-component will then increase as desired. However, a phase angle error is necessarily introduced as explained in Section 2.5. The $d$-current error increases as a consequence. Having passed through the $d$-signal limiter L2, the likely effect is just an angular displacement of the reference vector $u^*$ within the far-outside region of the actual bang-bang sector, Fig. 9. It is the error integrator in the I-channel which then accumulates the $d$-axis error, so that $u^*$ eventually enters an adjacent bang-bang sector. Only then is a change of the switching state commanded.

A lower voltage demand than that of the six-step mode initiates overmodulation. The stator current trajectory is then intermittently composed of sections of the six-step trajectory Fig. 13. Also here must a high $d$-axis current error be tolerated. The signal limiter L2 inhibits the deadbeat channel to interfere with the overmodulation strategy of the modulator.

$3.5$ Magnetic saturation

The unmodeled disturbances, which are predominantly low-frequency signals, do not impair the dynamic performance. However, magnetic saturation does have an effect on the deadbeat performance. The simplified model (16) depends on the inductance tensor (17), the parameters of which may undergo fast changes when the operating point changes. A saturation model is therefore indispensable. The saturation model consists of a table containing the components of the inductance tensor as function of the fundamental stator current. It is important that the saturation condition (17) of the simplified model gets updated in every modulation cycle. The table itself is created during a self-commissioning procedure.

The strongly nonlinear magnetic saturation characteristic may cause undesired current overshoots if the applied stator voltage is little higher than the required value. Such situation is likely to occur when a positive, large-signal change of the current is commanded. An overshoot is favored by the low inductance values of the machine, which enable current changes of rated magnitude within one subcycle. The inductances may then change in a ratio of 1:6. It is almost impossible in a practical application to predetermine the exact volt-seconds value required to achieve deadbeat response within a subcycle. The imperfections of the simplified model may also contribute to the error at large-signal excitation, but not decisively.

A simple, but effective cure to this problem is reducing the gain of the deadbeat controller by 10 - 20%. Saturation induced overshoots are thus avoided at the expense of a residual current error. This error gets almost compensated during the subsequent modulation interval. This is no penalty as the switching frequency is very high.

$4.$ Experimental Results

The control system was implemented in a standard 32-bit RISC microcontroller. The data of the PM synchronous mo-
tor are: $I_R = 2.8 \text{ A}$, $T_R = 4.6 \text{ Nm}$, $2p = 6$, $\omega_R = 3000 \times 2\pi/60 \ \text{sec}^{-1}$. The inverter operates at $f_s = 8 \text{ kHz}$ and $U_d = 300 \text{ V}$. Executing time for PWM is 5 $\mu$s, and 35 $\mu$s for current control including all I/O operations. The current regulator is clocked at 16 kHz.

Fig. 14 demonstrates the performance of the closed loop current control system in two steady-state overmodulation modes [10]. The unavoidable current waveform distortions are seen not to interfere with the modulation process.

Responses to a current step change of rated magnitude are shown in Fig. 15. The comparison demonstrates that the near-deadbeat controller compensates the deficiencies of the simplified machine model, while a regular PI controller produces higher rise time, overshoot, poor damping, and $d-q$ crosstalk. Fig. 16 demonstrates how dynamic overmodulation invokes bang-bang control to achieve the steepest rise of the torque building $q$-axis current. The penalty is a transient deviation of the $d$-axis current from its reference. The $d$-axis error is corrected during the next subcycle.

The anti-windup performance at a large-signal speed change is shown in Fig. 17(a). The $q$-axis current tracks its reference value with minimum delay. The high-frequency oscillations following the transition to the higher speed level are precisely executed as commanded by the speed controller. The phase currents of this process are displayed in Fig. 17(b) in an expanded time scale. Minimum overshoot can be observed. The near-deadbeat response is clearly visible at the trailing edge of $i_B$: The stair-cased waveform demonstrates that the speed controller is clocked at 250 $\mu$s. The dc-link voltage, $u_d$ in Fig. 17(a), sags during the acceleration. The dc-link capacitor is subsequently recharged by the 100-Hz pulses of the feeding single-phase bridge rectifier.

5. SUMMARY

The paper describes a control system for low-inductance positioning drives operating at high switching frequency. A pulsewidth modulator and a robust, high-performance current control system are implemented in standard microcontroller hardware. To minimize the computing requirements, the pulse sequence is generated by decision-making. Five different modulation modes cover the modulation range from space vector modulation to six-step operation, enabling precise current control in the entire overmodulation region.

A near-deadbeat controller generates defined volt-second increments at dynamic operation. An added PI controller channel is provided in parallel to maintain the small-signal stability of the system. The PI controller is relieved from large-signal changes and hence can be optimized for fast disturbance rejection.

The concept makes an accurate machine model obsolete; neither is the use of state or parameter observers required. The simplified model is represented by just an integrator. Magnetic saturation and their spatial variation is modeled in terms of $d$- and $q$-axis components.

The algorithms for modulation and control are designed and optimized for high hardware utilization. The sampling rate of the current control system is 16 kHz. There is about 35% of the processor time still available for superimposed controls, such as speed, position, and process control. Alternatively, the switching frequency can be increased. Very high switching frequency can be obtained by implementing the decision algorithms in a FPGA.

APPENDIX

The modulation index is the normalized fundamental voltage, defined as

$$m = \frac{u_1}{u_{1 \text{ six-step}}}, \quad (A-1)$$

where $u_1$ is the fundamental voltage of the modulated switching sequence and $u_{1 \text{ six-step}} = 2u_d/\pi$ the fundamental voltage
The definition of the distortion factor is derived from the rms harmonic current

\[ I_{h \text{ rms}} = \sqrt{\frac{1}{T} \int_T [i(t) - i_1(t)]^2 \, dt} \]  

which does not only depend on the performance of the pulse-width modulator, but also on the machine impedance, approximated by the leakage inductance \( l_\sigma \). This influence is eliminated when the distortion factor is used as a figure of merit. The distortion factor is derived from the normalized rms harmonic current

\[ \frac{I_{h \text{ rms}}}{I_1} = \frac{1}{T} \sqrt{\sum_{v=2}^{\infty} I_v^2} = \frac{\omega_1 l_\sigma}{U_1} \sqrt{\sum_{v=2}^{\infty} \left( \frac{U_v}{\omega_1 l_\sigma} \right)^2}, \]  

where \( I_1 \) and \( U_1 \) are the rms fundamental components, \( I_v \) and \( U_v \) the rms harmonic components of the machine current and voltage, respectively, and \( \omega_1 \) is the fundamental frequency. Assuming that the machine is supplied by an inverter operated in the six-step mode, the harmonic voltage components \( U_v \) can be easily determined from the rectangular voltage waveform, and we have from (A-3)

\[ \frac{I_{h \text{ rms six-step}}}{I_1} = 0.0464 \]  

Dividing (A-3) by (A-4) yields the distortion factor

\[ d = \frac{I_{h \text{ rms}}}{I_{h \text{ rms six-step}}}. \]  

The distortion factor is a figure of merit that characterizes the PWM quality. It does not depend on machine parameters.

6. REFERENCES